Sahlqvist Theory for Hybrid Logics (Unified Correspondence IV)

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Hybrid logics extends modal logic with a special sort of variables, called *nominals*, which are evaluated to singletons in Kripke models by valuations, thus acting as names for states in models, see e.g., [1], [2] and [7]. Various other connectives which capitalize on this naming power of the nominals and, in so doing, enhance the expressive power of these languages even further, can be included, most characteristically the *satisfaction operator*, $@_{\mathbf{i}}\varphi$, allowing one to express that φ holds at the world named by a nominal $\mathbf{i}$.

A natural research direction is to investigate the transfer of results from modal logic to hybrid logics. Indeed a lot of work has been done in this direction. One may, for example, consider Sahlqvist theory, which is extremely well developed for modal logic. The reader will recall that every modal Sahlqvist formula enjoys two properties: firstly, it has a local-first order frame correspondent and, secondly, it is canonical. The second property implies that any normal modal logic axiomatized with Sahlqvist formulas (in addition to the axioms of the basic modal logic $\mathbf{K}$) is strongly complete with respect to its Kripke frames.

As regards Sahlqvist theory for hybrid logic, it is fairly straightforward to see that nominals may be freely introduced into modal Sahlqvist formulas without destroying the first property. The second property is more tricky. In this regard, Ten Cate, Marx and Viana [10] showed that any hybrid logic obtained by adding *modal* Sahlqvist formulas to the basic hybrid logic $\mathbf{H}$ (the logic obtained by adding the axiom $\Diamond^n(\mathbf{i} \wedge p) \rightarrow \Box^m(\mathbf{i} \rightarrow p)$, $n, m \in \mathbb{N}$, and the rule that $\vdash \neg\mathbf{i}$ implies $\vdash \bot$ to the basic modal logic $\mathbf{K}$) is strongly complete. Also, one of the very first results in the study of hybrid logic was the fact that any extension of $\mathbf{H}$ with pure axioms (formulas containing no propositional variables but only possibly nominals) is strongly complete [7]. In [10], it is shown that these two results cannot be combined in general, since there is a modal Sahlqvist formula and a pure formula which together give a Kripke-incomplete logic when added to $\mathbf{H}$.

The intention of the present work is to see to what extent these two results *can* be combined and to develop a genuinely *hybrid* Sahlqvist theory. Some initial results in this direction appear in [3]. We define a hybrid version of the inductive formulas [8], encompassing both the modal Sahlqvist formulas and the pure formulas. In addition, we define two subclasses, called the *nominally skeletal* and *skeletal hybrid inductive formulas*. We show that members of these subclasses are respectively preserved under canonical extensions and Dedekind-MacNeille completions of certain *hybrid algebras*, which is enough to ensure that these formulas axiomatize relationally complete logics. Hybrid algebras were recently introduced in [9] as an algebraic semantics for hybrid logic, and greatly facilitate the reasoning in this work.

The key methodological tool in proving the above results is a hybrid version of the ALBA algorithm [5], which we formulate and call hybrid-ALBA. This algorithm manipulates formulas by applying a calculus of rewrite rules. In line with the philosophy of unified correspondence theory [4], these rules are entirely predicated upon the order-theoretic behaviour of the connectives. However, applying this philosophy in the setting of hybrid algebras requires certain innovations. In particular, the linchpin of the canonicity strategy employed in works like [4] and [5], namely the equivalence of validity and admissible validity of pure formulas, fails in this setting. This necessitates an investigation of the preservation of pure inequalities under completions of hybrid algebras. Other innovations include the use of a new algebraic semantics for hybrid logic based on hybrid algebras.

The work we will report on in this talk will appear in [6].

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