Using Topological Systems To Create a Framework for Institutions

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Abstract. Recently, J. T. Denniston, A. Melton, and S. E. Rodabaugh introduced a lattice-valued analogue of the concept of institution of J. A. Goguen and R. M. Burstall, comparing it, moreover, with the (lattice-valued version of the) notion of topological system of S. Vickers. This presentation shows that a suitable generalization of topological systems makes a convenient setting for doing (lattice-valued) institutions.

Keywords: institution, topological institution, topological system

There exists a convenient approach to logical systems in computer science, which is based in the notion of *institution* of J. A. Goguen and R. M. Burstall [8]. An institution is made by a category of (abstract) signatures, where every signature has its associated sentences, models, and a relationship of satisfaction. The latter relationship is invariant (in a certain sense) under change of signature. The slogan, therefore, is "truth is invariant under change of notation". Examples of institutions include, in particular, unsorted universal algebra, many-sorted algebra, order-sorted algebra, several variants of first-order logic, and partial algebra (see, e.g., [7]). Subsequently, a number of authors, including Goguen and Burstall, proposed various generalizations of institutions, while further advancing the theory [9, 10, 12–14]. Moreover, some of these authors worked within a purely category-theoretic approach to institutions (see, e.g., [6]).

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It is helpful to note the concept of *topological system* of S. Vickers [19], based in the ideas of geometric logic [20], provides a common setting for both topological spaces (point-set topology) and their underlying algebraic structures—locales (point-free topology). A topological system comprises a set, a locale, and a binary satisfaction relation between the elements of the set and members of the locale; an example of a topological system is the points and open sets of a topological space, together with the membership relation between them. In particular, S. Vickers presented system spatialization and localification functors, which opened ways to move back and forth between each pair of categories of topological spaces, locales, and topological systems.

Recently, the concept of topological system has gained interest in connection with lattice-valued topology. For instance, [3, 4] introduced and studied the notion of lattice-valued topological system; [11] discovered a convenient relationship between crisp and lattice-valued topology, based in topological systems; and [17, 18] studied a lattice-valued analogue of the system spatialization functor.

At the 35th Linz Seminar on Fuzzy Set Theory, J. T. Denniston, A. Melton, and S. E. Rodabaugh demonstrated relationships between institutions and topological systems by presenting a lattice-valued analogue of institutions, and they showed that (lattice-valued) topological systems provide a particular instance of the latter [5]. Moreover, [16] introduced (crisp) *topological institutions*, based in topological systems, with the slogan that "the central concept is the theory, not the formula". To continue this study, several authors considered other modifications of institutions (e.g., probability institutions, quantum institutions, etc. [1, 2]), motivated by the ideas of quantum logic in connection with quantum physics.

The main purpose of this presentation is to show that a suitably generalized concept of topological system provides a setting for a certain type of (lattice-valued) institutions, namely, elementary institutions of [15, 16].

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