Unified Correspondence as a Proof-Theoretic Tool (Unified Correspondence VII)

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This presentation reports on the results of [11], and focuses on the formal connections between correspondence phenomena, well known from the area of modal logic, and the theory of display calculi, originated by Belnap [1].

Sahlqvist correspondence theory. Sahlqvist theory [15] is among the most celebrated and useful results of the classical theory of modal logic, and one of the hallmarks of its success. It provides an algorithmic, syntactic identification of a class of modal formulas whose associated normal modal logics are *strongly complete* with respect to *elementary* (i.e. first-order definable) classes of frames.

Unified correspondence. In recent years, building on duality-theoretic insights [7], an encompassing perspective has emerged which has made it possible to export the state-of-the-art in Sahlqvist theory from modal logic to a wide range of logics which includes, among others, intuitionistic and distributive lattice-based (normal modal) logics [5], non-normal (regular) modal logics [14], substructural logics [6], hybrid logics [9], and mu-calculus [2,3].
The breadth of this work has stimulated many and varied applications. Some are closely related to the core concerns of the theory itself, such as the understanding of the relationship between different methodologies for obtaining canonicity results [13], or of the phenomenon of pseudocorrespondence [8]. Other, possibly surprising applications include the dual characterizations of classes of finite lattices [10]. These and other results have given rise to a theory called *unified correspondence* [4].

Tools of unified correspondence theory. The most important technical tools in unified correspondence are: (a) a very general syntactic definition of the class of Sahlqvist formulas, which applies uniformly to each logical signature and is given purely in terms of the order-theoretic properties of the algebraic interpretations of the logical connectives; (b) the algorithm ALBA, which effectively computes first-order correspondents of input term-inequalities, and is guaranteed to succeed on a wide class of inequalities (the so-called *inductive* inequalities) which, like the Sahlqvist class, can be defined uniformly in each mentioned signature, and which properly and significantly extends the Sahlqvist class.

Unified correspondence and display calculi. The proposed talk focuses on an entirely different type of application of unified correspondence: the identification of the syntactic shape of axioms which can be translated into analytic rules of a display calculus. A rule is called *analytic* if adding it to a display calculus preserves Belnap's cut-elimination theorem. The connections between Sahlqvist theory and display calculi have been seminally observed by Marcus Kracht in [12], in the context of his characterisation of those formulas of the language of basic modal logic (which he calls *primitive formulas*) which can be effectively transformed into structural rules of display calculi.

Contributions. The two tools of unified correspondence can be put to use to generalise Kracht's transformation procedure from axioms into analytic rules. This generalisation concerns more than one aspect. Firstly, in the same way in which the definitions of Sahlqvist and inductive inequalities can be given uniformly in each logical signature, the definition of primitive formulas/inequalities is introduced for any logical framework the algebraic semantics of which is based on distributive lattices with operators. Secondly, in the context of each such logical framework, we introduce a hierarchy of subclasses of inductive inequalities, progressively extending the primitive inequalities, the largest of which is the class of so-called *analytic inductive inequalities*. This class significantly generalises the class of primitive formulas/inequalities. We provide an effective procedure, based on ALBA, which transforms each analytic inductive inequality into an equivalent set of analytic rules. Moreover, we show that any analytic rule can be effectively and equivalently transformed into some analytic inductive inequality.

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