An algebraic approach to Probabilistic Dynamic Epistemic Logic

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Probabilistic DEL. The family of Probabilistic Dynamic Epistemic Logics (PDELs) consists of expansions of the well known dynamic epistemic logic in which the epistemic uncertainty of agents—as well as changes caused by actions and events—is modelled both qualitatively (i.e. by means of epistemic modal operators) and quantitatively (i.e. by means of probability distributions). Various PDEL-systems exist, such as the one introduced in [8], which is designed to encode three types of probability: prior probability on states, occurrence probabilities in the relevant process taking place, and observation probabilities of events (which are observed by each agent from her own viewpoint). The update mechanism of [4] is then generalized and adapted to account for the interaction of these three types of probabilities, and a complete axiomatization is introduced. Other variants include [3] and [1], and have been employed to model the phenomenon of informational cascades, one of the standard examples of which is the *urn example*, presented below following [3], to formally analyze a certain kind of situations in which individual rationality may lead to "group irrationality".

Informational cascades: the urn example. Consider two urns, U_W and U_B, such that U_W contains two white balls and one black ball, and U_B contains one white ball and two black balls. One urn is randomly picked and placed in a room. This setup is common knowledge to a group of agents $a_1, a_2, ..., a_n$ but they do not know which urn is in the room. The agents enter the room one at a time; first a_1, then a_2, and so on. Each agent draws one ball from the urn, looks at it, puts it back, and leaves the room. Hence, only the person in the room knows which ball she drew. After leaving the room each agent makes a guess as to whether it is urn U_W or U_B that is placed in the room and writes her guess on a blackboard for all the other agents to see. Therefore, each agent knows the guesses of the agents preceding her in the sequence before entering the room herself. It is common knowledge that they will be individually rewarded if and only if their own guess is correct.
Let us assume that U_B is the urn placed in the room. When a_1 enters and draws a ball, if she draws a white ball it is rational to make a guess for U_W, whereas if she draws a black one she should guess U_B. Moreover, when making a guess for U_W (resp. U_B), all the other agents can infer that she drew a white (resp.

black) ball. When a_2 enters the room after a_1, she knows the color which a_1 drew, and it is obvious how she should guess if she draws a ball of the same color. If a_2 draws a ball of opposite color than a_1, then the probabilities for U_W and U_B become equal. Assume for simplicity that that any individual faced with equal probability for U_W and U_B will guess for the urn that contains more balls of the color she saw herself, and that this tie-breaking rule is common knowledge among the agents. When a_3 enters, a cascade can arise. Indeed, if a_1 and a_2 drew the same color of balls (given the reasoning previously described, a_3 will know this), say both white, then no matter which color of ball a_3 draws, the posterior probability of U_W will be higher than the probability of U_B. So if the agent a_3 is rational, she will write U_W no matter which ball she drew. The agents following a_3 should therefore take a_3's guess as conveying no new information. Furthermore, everyone after a_3 will have the same information as a_3 (the information about what a_1 and a_2 drew). Hence, their reasoning will be identical to a_3's, which explains the cascade leading to everyone making the same (wrong) guess.

Probabilistic updates on algebras. Our contribution aims at extending the PDEL framework to its counterparts on a weaker than classical propositional base. Following the methodology introduced in [5,6] and further developed in [7,2], we obtain the dual characterization of the probabilistic update construction, in the environment of complete atomic BAOs equipped with subjective probability measures on subsets of the algebras. This characterization readily generalizes to much wider classes of models based on algebras (such as arbitrary Boolean or Heyting algebras) equipped with modal operations and probability measures, and hence results in a point-free version of the probabilistic updates on set-based models. As in [5,6], this update construction makes it possible to define a semantic interpretation of the language of PDEL on these algebraic models. This interpretation in its turn makes it possible to introduce a semantically motivated axiomatization of PDEL on intuitionistic propositional base.

Work in progress. We plan to use this framework to describe social and epistemic situations such as informational cascades in the setting of nonclassical logics in which the law of excluded middle fails. For instance, in the case of the urn example, the procedure to establish whether the urn is U_W or U_B could itself follow a majority rule, and hence in case of a draw, the truth value might remain undetermined. Truth-establishing procedures such as this one would more faithfully model the reasoning dynamics in groups of agents where benefit is derived jointly from correctly determining the truth of some proposition and by having one's decision in this regard coincide with that of the majority. This situation is relevant to many concrete settings studied in social science and management science. For example, when deciding whether to buy the work of an artist, a collector will form an opinion about the quality of the work. Many collectors will also think about the opinions of other collectors and the likelihood that they would want to acquire work by this artist now and in the future. After all, if few others see, or come to see, value in this work, it will not be an investment from which one can profit, either financially or in terms of prestige. This is even more clearly the case when an investor buys a stock for speculative purposes, to sell it for a profit in the short run.

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