Canonicity and Relativized Canonicity via Pseudo-Correspondence (Unified Correspondence II)

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Unified correspondence theory. The contributions reported on in the proposed talk pertain to *unified correspondence theory* [3], a line of research which applies duality-theoretic insights to Sahlqvist theory (cf. [6]), with the aim of uniformly extending the benefits of Sahlqvist theory from modal logic to a wide range of logics which include, among others, intuitionistic and distributive lattice-based (modal) logics [4], regular modal logics [14], substructural logics [5], hybrid logics [8], and mu-calculus [1,2]. Applications of unified correspondence are very diverse, and include the understanding of the relationship between different methodologies for obtaining canonicity results [13], the dual characterizations of classes of finite lattices [10], and the identification of the syntactic shape of axioms which can be translated into structural rules of a properly displayable calculus [11].

Contributions. The proposed talk reports on the results in [7]. In this paper, the tools of unified correspondence are applied to provide a canonicity result, *without* an accompanying elementarity result, for a wide class of axioms of distributive lattice-based logics. Specifically, we generalize Venema's pseudo-correspondence argument for the canonicity of the additivity of positive terms [16] in the generalized setting of distributive lattice expansions, using the rules of the algorithm/calculus for correspondence ALBA (cf. [4]) and the methodology of unified correspondence.
The order-theoretic facts underlying this generalization provide the basis for the soundness of additional ALBA rules relative to the classes of structures in which the formulas asserting the additivity of some given terms are valid. These classes do not need to be first-order definable, and in general they are not. Accordingly, an enhanced version of ALBA, which we call ALBA^e, is defined, which is proven to be successful on a certain class of inequalities which significantly extends the class of inequalities on which the canonicity-via-correspondence argument is known to work (see Section 3 in [4]). These inequalities are shown to be canonical relative to the subclass defined by the given additivity axioms.

Relevance to other research themes. These results contribute to the exploration of canonicity in the presence of additional axioms (or relativized canonicity). It is well known that certain modal axioms which are *not* in general canonical (i.e., over the class of all algebras) *are* canonical over some smaller class of algebras. Examples of relativized canonicity are rather rare, canonicity in the presence of transitivity being one example: in [12], Lemmon and Scott prove that the McKinsey formula becomes canonical when taken in conjunction with the transitivity axiom. Moreover, all modal reduction principles are canonical in the presence of transitivity, and this can be seen as follows: in [17] Zakharyaschev proves that any extension of **K4** axiomatized with modal reduction principles has the finite model property, and is hence Kripke complete. Combining this fact with the elementarity of the reduction principles over transitive frames as proved by van Benthem [15], the claim follows by Fine's theorem [9]. The problem of relativized canonicity is difficult to tackle directly and for classes of axioms. The results discussed in the proposed talk can be regarded as a contribution in this direction.

Also, the new rules of the enhanced ALBA, as well as the generalized canonicity-via-correspondence argument (which is based on a new, 'conditional' version of Esakia lemma), form the technical basis for the extension of unified correspondence theory to regular modal logics. This is the focus of [14], discussed in a companion proposal for a contributed talk.

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