Dual characterizations for finite lattices via correspondence theory for monotone modal logic (Unified Correspondence VI)

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Dual characterization results for finite lattices. The proposed talk reports on the paper [10]. The starting point of this paper is a duality for finite lattices, established by Santocanale [17]. The structures dually equivalent to finite lattices are referred to as *join-presentations*, and are certain triples $(X, \leq, \mathcal{M})$ such that $(X, \leq)$ is a finite poset, and $\mathcal{M} : X \longrightarrow \mathcal{P}\mathcal{P}X$. In [17], it has been pointed out—and indicated as a worthwhile research direction—that the existence of this duality makes it possible to investigate systematic *dual characterization* results, between equations or inequalities in the algebraic language of lattices on one side, and first-order conditions in the language of join-presentations on the other. One significant instance of such systematic dual characterizations has been developed in the same paper, between a class of inequalities in the language of lattices and a corresponding class of first-order conditions. This result generalizes Nation's [14, Section 5] stating that a certain class of finite lattices[1] is a pseudovariety, and is similar to Semenova's results in [18].

From modal logic to unified correspondence theory. Modal logic is an area in which systematic dual characterization results have been extensively developed, giving rise to a very rich theory—the so-called *modal correspondence theory*—which has been investigated for almost forty years. Modal correspondence theory was originally developed in a purely model-theoretic way [1]. However, correspondence-related phenomena have been studied in an algebraic framework subsuming duality theory since the early 50s [13], and very recently, building on [4,5], a *unified correspondence* framework has emerged [3], which is based on duality, uniformly extends correspondence theory to an array of nonclassical logics which includes (distributive) lattice-based modal logic [4,8], regular modal logics [16], modal mu-calculus [7,2], and hybrid logic [6], and has been applied to different issues, including the understanding of the relationship between different methodologies for obtaining canonicity results [15], or of the phenomenon of pseudo-correspondence [9]. and the identification of the syntactic shape of axioms which can be translated into structural rules of a properly displayable calculus [11]. The present talk concerns another such application of the tools of unified correspondence.

[1] Namely, the finite lattices such that the length of their *D-chains* has a uniform upper bound.

Main contribution. Indeed, given the availability of this theory, it seems natural to try and understand dual characterization results such as [17, Proposition 8.5] as instances of a more general unified correspondence mechanism. This is what the paper [10] does, by establishing a novel dual characterization result similar to Nation's. Our result paves the way to the mechanization and systematization of dual characterizations such as the one in [17].

Methodology: basic algorithmic correspondence for monotone modal logic. Our approach is based on an adaptation of the algorithm/calculus ALBA of [4] to the case of *monotone modal logic*. This adaptation is necessary, since some of the rules in the standard version of the algorithm would not be sound for the modal connectives of monotone modal logic, and is one of the contributions of [10]. The adapted ALBA is semantically justified in the general environment of two-sorted frames, which are general structures that can encode monotone neighbourhood frames as special cases. As their name suggests, two-sorted frames are relational structures based on two domains. Normal modal operators can be associated in the standard way with the binary relations on two-sorted frames. Monotone modal operators can be then interpreted on two-sorted frames as the composition of some of these normal modalities. This provides the basic semantic environment for the adapted ALBA.
Correspondence theory for monotone modal logic has already been studied in [12], where a class of monotone modal formulas which are guaranteed to have a first-order correspondent has been identified. However, the class of inductive inequalities corresponding to the ALBA setting is strictly larger than the one in [12].

Enhancing ALBA. However, the translations of inequalities such as Nation's [14] and as the ones treated in the present paper fall outside the inductive class. Hence, another contribution of [10] is the addition of special rules which are sound on the specific semantic setting arising from finite lattices. Interestingly, an Ackermann-type rule features among these additional rules, the soundness of which cannot be straightforwardly explained in terms of the Ackermann lemma (which, as discussed in [4,3], is the engine of the original ALBA), but which however still intuitively encodes a minimal valuation argument.

References

1. van Benthem, J.: Modal Logic and Classical Logic. Indices : Monographs in Philosophical Logic and Formal Linguistics, Vol 3, Bibliopolis (1985), http://books.google.co.za/books?id=j8AVAQAACAAJ
2. Conradie, W., Fomatati, Y., Palmigiano, A., Sourabh, S.: Algorithmic correspondence for intuitionistic modal mu-calculus. Theoretical Computer Science 564, 30–62 (2015), http://dx.doi.org/10.1016/j.tcs.2014.10.027
3. Conradie, W., Ghilardi, S., Palmigiano, A.: Unified correspondence. In: Baltag, A., Smets, S. (eds.) Johan van Benthem on Logical and Informational Dynamics. Outstanding Contributions to Logic, Springer (2014)

4. Conradie, W., Palmigiano, A.: Algorithmic correspondence and canonicity for distributive modal logic. Annals of Pure and Applied Logic 163(3), 338 – 376 (2012), http://www.sciencedirect.com/science/article/pii/S016800721100162X
5. Conradie, W., Palmigiano, A., Sourabh, S.: Algebraic modal correspondence: Sahlqvist and beyond. Submitted
6. Conradie, W., Robinson, C.: On Sahlqvist theory for hybrid logic. Journal of Logic and Computation (forthcoming)
7. Conradie, W., Craig, A.: Canonicity results for mu-calculi: an algorithmic approach. Journal of Logic and Computation (forthcoming)
8. Conradie, W., Palmigiano, A.: Algorithmic correspondence and canonicity for non-distributive logics. Journal of Logic and Computation (forthcoming)
9. Conradie, W., Palmigiano, A., Sourabh, S., Zhao, Z.: Canonicity and relativized canonicity via pseudo-correspondence: an application of ALBA. Submitted (2014)
10. Frittella, S., Palmigiano, A., Santocanale, L.: Dual characterizations for finite lattices via correspondence theory for monotone modal logic. Journal of Logic and Computation (forthcoming)
11. Greco, G., Ma, M., Palmigiano, A., Tzimoulis, A., Zhao, Z.: Unified correspondence as a proof-theoretic tool. Submitted (2014)
12. Hansen, H.: Monotonic Modal Logics. Master's thesis, University of Amsterdam, http://www.illc.uva.nl/Publications/ResearchReports/PP-2003-24.text.ps.gz (2003)
13. Jónsson, B., Tarski, A.: Boolean algebras with operators Part I. Amer. J. Math. (73), 891–939 (1951)
14. Nation, J.B.: An approach to lattice varieties of finite height. Algebra Universalis 27(4), 521–543 (1990), http://dx.doi.org/10.1007/BF01188998
15. Palmigiano, A., Sourabh, S., Zhao, Z.: Jónsson-style canonicity for ALBA-inequalities. Journal of Logic and Computation (forthcoming)
16. Palmigiano, A., Sourabh, S., Zhao, Z.: Sahlqvist theory for impossible worlds. Journal of Logic and Computation (forthcoming)
17. Santocanale, L.: A duality for finite lattices (2009), https://hal.archives-ouvertes.fr/hal-00432113/PDF/0.pdf
18. Semenova, M.V.: On lattices that are embeddable into lattices of suborders. Algebra Logika 44(4), 483–511, 514 (2005), http://dx.doi.org/10.1007/s10469-005-0027-7