An Abstract Algebraic Logic view on Judgment Aggregation

María Esteban, Alessandra Palmigiano, and Zhiguang Zhao

Social choice and judgment aggregation. The theory of *social choice* is the formal study of mechanisms for collective decision making, and investigates issues of philosophical, economic, and political significance, stemming from the classical Arrovian problem of how the preferences of the members of a group can be "fairly" aggregated into one outcome.
In the last decades, many results appeared generalizing the original Arrovian problem, which gave rise to a research area called *judgment aggregation* (JA) [16]. While the original work of Arrow [1] focuses on preference aggregation, this can be recognized as a special instance of the aggregation of consistent judgments, expressed by each member of a group of individuals over a given set of logically interconnected propositions (the *agenda*): each proposition in the agenda is either accepted or rejected by each group member, so as to satisfy certain requirements of logical consistency. Within the JA framework, the Arrovian-type *impossibility results* (axiomatically providing sufficient conditions for aggregator functions to turn into degenerate rules, such as dictatorship) are obtained as consequences of *characterization theorems* [17], which provide necessary and sufficient conditions for agendas to have aggregator functions on them satisfying given axiomatic conditions.T In the same logical vein, in [15], *attitude aggregation theory* was introduced; this direction has been further pursued in [10], where a characterization theorem has been given for certain many-valued propositional-attitude aggregators as MV-algebra homomorphisms.
The ultrafilter argument and its generalizations. Methodologically, the *ultrafilter argument* is the tool underlying the generalizations and unifications mentioned above. It can be sketched as follows: to prove impossibility theorems for finite electorates, one shows that the axiomatic conditions on the aggregation function force the set of all decisive coalitions to be an (ultra)filter on the powerset of the electorate. If the electorate is finite, this implies that all the decisive coalitions must contain one and the same (singleton) coalition: the oligarchs (the dictator). First employed in [14] for an alternative proof of Arrow's theorem, this argument was applied to obtain elegant and concise proofs of impossibility theorems also in JA [3]. More recently, it gave rise to characterization theorems establishing a bijective correspondence between Arrovian aggregation rules and ultrafilters on the set of individuals [11] and between certain judgment aggregation functions and ultraproducts of profiles [8]. Using the correspondence between ultrafilters and Boolean homomorphisms, similar correspondences have been established between Arrovian judgment aggregators and Boolean algebra homomorphisms [9].
Escaping impossibility via nonclassical logics. While much research in this area explored the limits of the applicability of Arrow-type results, at the same time the question of how to 'escape impossibility' started attracting increasing interest. In [2], Dietrich shows that impossibility results do not apply to a wide class

of realistic agendas once propositions of the form 'if a then b' are modelled as *subjunctive* implications rather than material implications. Besides its theoretical value, this result is of practical interest, given that subjunctive implication models the meaning of if-then statements in natural language more accurately than material implication.

Aim. In the light of Dietrich's result, how can we highlight the role played by the logic (understood both as formal language and deductive machinery) underlying the given agenda in characterization theorems for JA? The present talk focuses on *Abstract Algebraic Logic* (AAL) as a natural theoretical setting for Herzberg's results [8,10], and on *(fully) selfextensional logics* as the appropriate setting for a nonclassical interpretation of logical connectives, in line with [2].

Abstract Algebraic Logic and selfextensional logics. AAL is conceived as the framework for an algebraic approach to the investigation of classes of logics. *Selfextensionality* is the metalogical property holding of those logical systems whose associated relation of logical equivalence on formulas is a congruence of the term algebra. In [18], selfextensional logics are characterized as the logics which admit a general version of the well known possible-world semantics of modal and intuitionistic logics, and in [13], a characterization was given of the particularly well behaved subclass of the fully selfextensional logics in general duality-theoretic terms. This subclass includes many well-known logics, such as classical, intuitionistic, modal, many-valued and relevance logic. These and other results (cf. e.g. [12,7,4,5]) establish a systematic connection between possible world semantics and the logical account of intensionality.

Contributions. In [6], the characterization result of [10] is generalized and refined from MV-algebras to any class of algebras canonically associated with some selfextensional logic. In contrast to [10], the properties of agendas are independent of a specific logical signature, and the resulting characterization is symmetric. Aggregation of propositional attitudes modeled in classical, intuitionistic, modal, many-valued and relevance logic can be uniformly captured as special cases of the present result. This makes it possible to fine-tune the expressive and deductive power of the formal language of the agenda, so as to capture e.g. intensional or vague statements.

For any selfextensional logic $\mathcal{S}$, the *agenda* is a set of formulas $X \subseteq \mathbf{Fm}_{\mathcal{S}}$. An *attitude function* is a map $A \in \mathbf{B}^X$ assigning each formula in the agenda to an element of the $\mathcal{S}$-algebra $\mathbf{B}$. The *electorate* is a set N. An *attitude profile* is an element $\boldsymbol{A} \in (\mathbf{B}^X)^N$. For each $\varphi \in X$, let $\boldsymbol{A}(\varphi)$ denote $\{A_i(\varphi)\}_{i \in N} \in \mathbf{B}^N$. An *attitude aggregator* is a partial map $F : (\mathbf{B}^X)^N \nrightarrow \mathbf{B}^X$. An attitude function $A \in \mathbf{B}^X$ is *rational* if it can be extended to a homomorphism $\bar{A} : \mathbf{Fm}_{/\equiv} \longrightarrow \mathbf{B}$ of $\mathcal{S}$-algebras. A profile $\boldsymbol{A} \in (\mathbf{B}^X)^N$ is *rational* if A_i is a rational attitude function for each $i \in N$. An attitude aggregator $F : (\mathbf{B}^X)^N \nrightarrow \mathbf{B}^X$ is *rational* if $F(\boldsymbol{A})$ is a rational attitude function for all rational profiles $\boldsymbol{A} \in dom(F)$, and is *universal* if $dom(F) = (\mathbf{B}^X)^N$. A *decision criterion* for F is a partial map $f : \mathbf{B}^N \nrightarrow \mathbf{B}$ such that $F(\boldsymbol{A})(\varphi) = f(\boldsymbol{A}(\varphi))$ for all $\boldsymbol{A} \in dom(F)$ and all $\varphi \in X$. An aggregator F is *strongly systematic* if there exists some decision criterion f for F such that $F(\boldsymbol{A})(\varphi) = f(\boldsymbol{A}(\varphi))$ for all $\varphi \in \bar{X}$ and $\boldsymbol{A} \in dom(F)$.

Proposition 1. *For any rational, universal and strongly systematic attitude aggregator F, the decision criterion of F is a homomorphism of $\mathcal{S}$-algebras. Conversely, any homomorphism $f : \mathbf{B}^N \nrightarrow \mathbf{B}$ of $\mathcal{S}$-algebras gives rise to a rational, universal and strongly systematic attitude aggregator $F : (\mathbf{B}^X)^N \nrightarrow \mathbf{B}^X$, defined by $F(\boldsymbol{A})(\varphi) = f(\boldsymbol{A}(\varphi))$ for any rational profile $\boldsymbol{A}$ and any $\varphi \in X$.*

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