Comparing presentations of algebraic theories

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(Classical) equational theories, abstract clones, (finitary) monads, Lawvere theories, and operads (with actions of finite functions) are five different means of describing algebraic structures. In the category of sets these different means describe the same algebraic structures. In fact, suitably defined categories of equational theories, abstract clones, finitary monads, Lawvere theories, and operads are equivalent.

There are various 'subclasses' of equational theories that are better behaving from certain points of view. Regular theories are axiomatized by equations having the same variables on both sides (e.g. sup-semilattices), linear-regular theories are axiomatized by equations having the same variables on both sides each occurring exactly once (e.g. commutative monoids), and rigid equational theories are those linear regular theories that do not prove any equation of form $t(x_1, \ldots, x_n) = t(x_{\sigma(1)}, \ldots, x_{\sigma(1)})$ for any term t in n-variables and permutation $\sigma \in S_n$ (e.g. monoids with anti-involution).

Such theories have a 'geometric' content with the last two kinds of theories widely used in combinatorics, particularly in the form of either monads (analytic and polynomial) or operdas (symmetric and rigid). Lawvere theories in the corresponding subclasses have also elegant categorical characterizations via existence of some factorization systems. The conditions identifying these classes using abstract clones seem to be less natural.

In the talk I will discuss how the global correspondence between different presentations of theories restricts to the mentioned subclasses.

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