Category theory and Homotopy type theory

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Homotopy type theory has a variety of goals, including the foundation of mathematics (the *constructive foundation* of Martin-Löf or the *univalent foundation* of Voevodsky). More prosaically, it can be used as an axiomatic system for the homotopy theory of topological spaces (or of Kan complexes). The system comes in many variants depending on the syntax, the deduction rules and the axioms. Our goal is to develop homotopy type theory conceptually and invariantly. For this we use the language of category theory, as in Quillen's homotopical algebra. Our central notion is that a *tribe*: it is category equipped with a class of maps called *fibrations* satisfying a few axioms. A tribe can have internal products or not, it can be simplicially enriched or not. Every tribe has the structure of a fibration category in the sense of Ken Brown. We introduce the notion of homotopy equivalence between tribes: equivalent tribes have the same *content*. We also introduce a notion of fibration between tribes, called *meta-fibration*. We show that the category of simplicial tribes has the structure of a Brown fibration category. It is a step toward the homotopy theory of homotopy type theories. It is related to the recent work of Karol Szumilo.