Hausdorff mapping invariance theorems with sublocales

Jorge Picado

CMUC, Department of Mathematics, University of Coimbra, Portugal
picado@mat.uc.pt

We shall illustrate the *art of point-free thinking* on sublocales with several results about normality type properties and their duals.

Sublocale lattices are much more complicated than their topological counterparts (i.e. complete, atomic Boolean algebras). One of the main differences is that only complemented sublocales (and most sublocales are not complemented) distribute over all joins of sublocales. But, as J. Isbell emphasized, 'a locale has enough complemented sublocales' to compensate for this shortcoming: one simply has to 'make the sublocales which are complemented do more of the work' [5].

This talk will report on recent results that nicely illustrate that idea [1–4]. First, using the covariant approach to localic maps and sublocales of [6], we extend the Hausdorff mapping invariance theorem of 1936 (that characterizes normal spaces as the spaces whose image under any closed continuous map is normal) to the localic setting. Our approach covers several fundamental variants of normality at once. For that, the notion of normality is stated relatively to a fixed class $\mathscr{A}$ of complemented sublocales of the given locale. Besides unifying a variety of well-known classical variants of normality, this has another feature: by taking complements in $\mathscr{A}$, one gets immediately dual results about extremal disconnectedness type properties. Other cases treated are e.g. the hereditary and perfectness properties. General (not necessarily continuous) real functions and their semicontinuity properties and insertion results fit then nicely in the picture.

This is joint work with J. Gutiérrez García (Bilbao). Parts of it are also joint work with T. Kubiak (Poznán) and A. Pultr (Prague).

References

1. M. J. Ferreira, J. Gutiérrez García and J. Picado: *Completely normal frames and real-valued functions.* Topology Appl. 156 (2009) 2932-2941.
2. J. Gutiérrez García, T. Kubiak and J. Picado: *Localic real functions: a general setting.* J. Pure Appl. Algebra 213 (2009) 1064-1074.
3. J. Gutiérrez García, T. Kubiak and J. Picado: *Extremal disconnectedness and the hereditary property*, in preparation.
4. J. Gutiérrez García and J. Picado: *On the parallel between normality and extremal disconnectedness.* J. Pure Appl. Algebra 218 (2014) 784-803.
5. J. Isbell: *Atomless parts of spaces.* Math. Scand. 31 (1972) 5-32.
6. J. Picado and A. Pultr: *Frames and locales: Topology without points.* Frontiers in Mathematics, vol. 28, Springer, Basel (2012).