Higher-Order Game Theory

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I wish to report here on a novel formalisation of Game Theory based on higher order functionals. The starting point is the modelling of players via so-called *selection functions*, i.e. functionals of type

$$(X \to R) \to X.$$

Here X is the type of moves and R is the type of outcomes. If one thinks of $X \to R$ as the type of possible *game contexts*, a selection function describes the optimal moves in each given game context. Some of the main results so far include:

- The type constructor $J_R X \equiv (X \to R) \to X$ is a strong monad, and as such supports an operation

  $$J_R X \times J_R Y \to J_R(X \times Y).$$

  This can be understood as a "merging" of players. The new selection function $J_R(X \times Y)$ is a new single player that captures the goals of the two given players $J_R X$ and $J_R Y$, see [3, 4].
- With an appropriate definition of *equilibrium* one can show that in sequential games the operation $\otimes$ calculates optimal strategies. Moreover, with $\mathrm{argmax}\colon (X \to \mathbb{R}^n) \to X$ as selection functions, as in standard Game Theory, this construction coincides with backward induction [6].
- The binary operation $\otimes$ can be iterated not only finitely many times, but also a countable number of times, i.e.

  $$\Pi_{i\in\mathbb{N}} J_R X_i \to J_R \Pi_{i\in\mathbb{N}} X_i$$

  is well-defined (assuming R a discrete type, and continuity of functionals) and in fact has been shown to be equivalent to bar recursion, a proof-theoretic construction used to give computational meaning to the countable axiom of choice [3].

Selection functions were first introduced in [1, 2] with $R = \mathbb{B}$, and later generalised in [3–6].

References

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