Finitely presented MV-algebras, unital lattice ordered abelian groups and rational polyhedra - together

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The celebrated Markov unrecognizability theorem shows the impossibility of equipping any combinatorial manifold M with a computable set of invariants characterizing M up to homeomorphism. To make sense of the statement of the theorem, M is typically replaced by a finite string of symbols representing a triangulated rational polyhedron P_M, and homeomorphism is understood as (rational) PL-homeomorphism. Viewing the recognizability problem from the viewpoint of algorithmic complexity theory, one may naturally take into account the amount of information needed to specify P_M. We are thus left with the category of rational polyhedra (objects) with integer PL-maps (arrows). A new geometry arises, where the affine group over the integers takes on the same role as the isometry group does in euclidean geometry. Many arithmetic geometric computable invariants emerge in this new category, which make no sense for rational polyhedra with rational PL-maps. We discuss in particular the rational measure of rational polyhedra. Its role and applicability is amplified by their duality with finitely presented MV-algebras, the algebras of finitely axiomatizable theories in Lukasiewicz infinite-valued logic.

References

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