Frames, topologies, and duality theory

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Lecture 4

Recap

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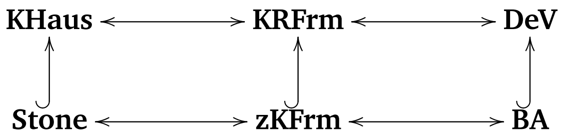

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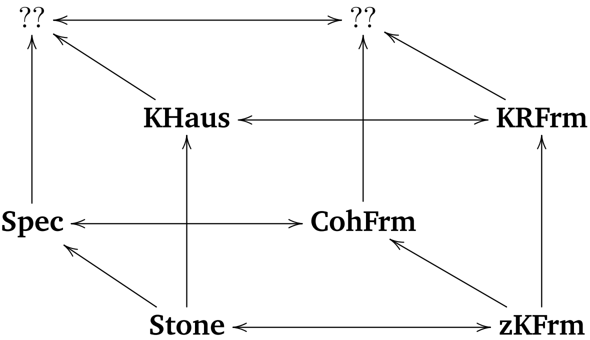

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We say that a is way below b and write $a \ll b$ if $b \leqslant \bigvee S$ implies $a \leqslant \bigvee T$ for some finite $T \subseteq S$.

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The correspondence: $F \mapsto \bigcap\{O(a) \mid a \in F\}$ and $K \mapsto \{a \in L \mid K \subseteq O(a)\}$.

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Proof: The restrictions of the contravariant functors Ω, pt to **StKSp** and **StKFrm**, respectively, yield the desired dual equivalence.

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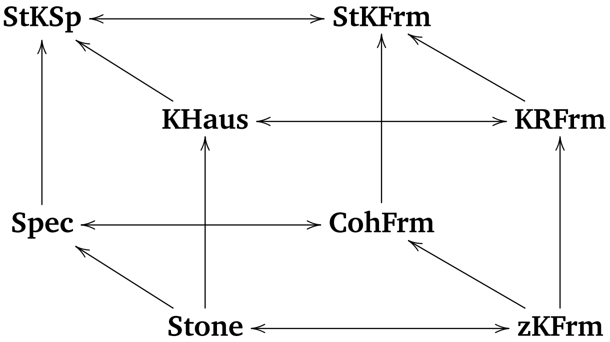

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This explains why stably compact spaces are the "right" generalization of compact Hausdorff spaces.

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A version of it for de Vries algebras is discussed in

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Another important construction on **KHaus** is the Vietoris hyperspace construction. Its pointfree analogue was developed by Johnstone.

P. T. Johnstone, *Vietoris locales and localic semilattices*, Continuous lattices and their applications (Bremen, 1982), Lecture Notes in Pure and Appl. Math. (1985), pp. 155–180.

A version of it for de Vries algebras is discussed in

G. Bezhanishvili, N. Bezhanishvili, J. Harding. *Modalities on compact regular frames and de Vries algebras*, Applied Categorical Structures, **23** (2015), pp. 365–379.

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J. D. Lawson, *Order and strongly sober compactifications*, Topology and category theory in computer science (Oxford, 1989), Oxford Sci. Publ., 1991, pp. 179–205.

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W. A. J. Luxemburg and A. C. Zaanen, *Riesz spaces. Vol. I*, North-Holland Publishing Co., Amsterdam, 1971.

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Long-standing open problem: Is every superintuitionistic logic topologically complete?

Thank you!