# Uniform Interpolation and Compact Congruences

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**Theorem (Pitts, 1992)**

*For any formula* $\phi(\overline{x}, \overline{y})$ *of intuitionistic propositional logic* IPC*, there exist* **left** *and* **right uniform interpolants***, i.e., formulas*

$$\phi^L(\overline{y}) \quad \textit{and} \quad \phi^R(\overline{y})$$

*of* IPC *with variables in* $\overline{y}$*, such that for any formula* $\psi(\overline{y}, \overline{z})$*,*

$$\phi(\overline{x}, \overline{y}) \vdash_{\mathrm{IPC}} \psi(\overline{y}, \overline{z}) \quad \Leftrightarrow \quad \phi^R(\overline{y}) \vdash_{\mathrm{IPC}} \psi(\overline{y}, \overline{z})$$

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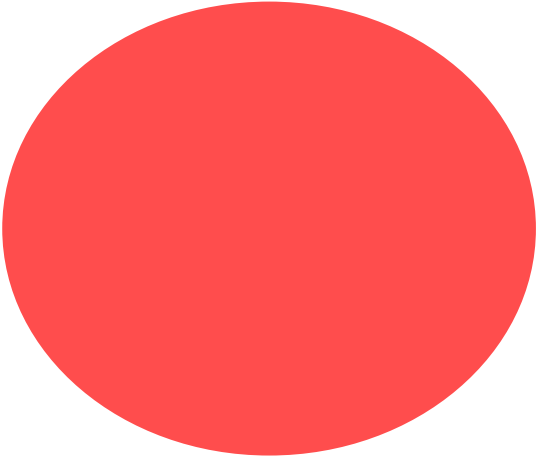
## Notation

The formula $\phi^L$ is often denoted by $\forall_{\overline{x}}\phi$ and $\phi^R$ by $\exists_{\overline{x}}\phi$.

# Right uniform interpolation

$\mathbf{Fm}(\overline{x}, \overline{y})$



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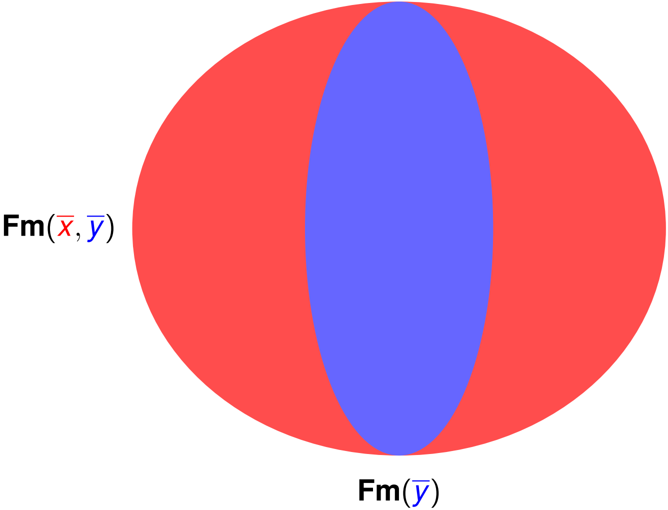

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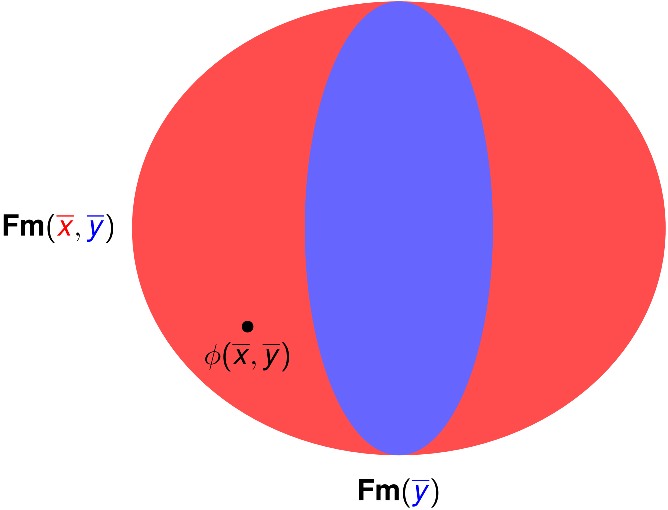

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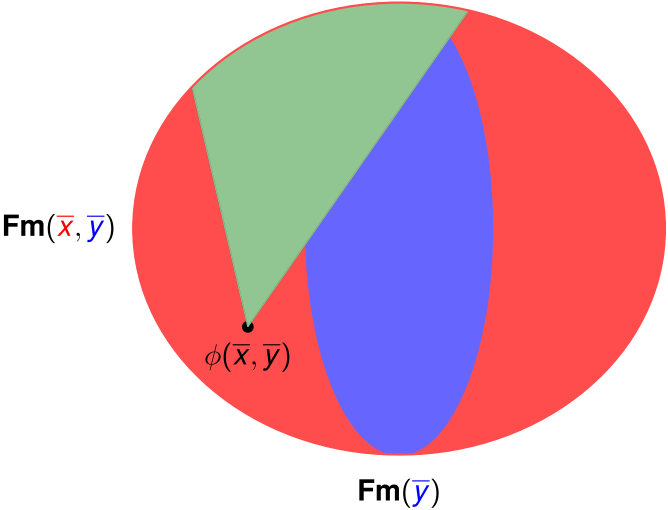

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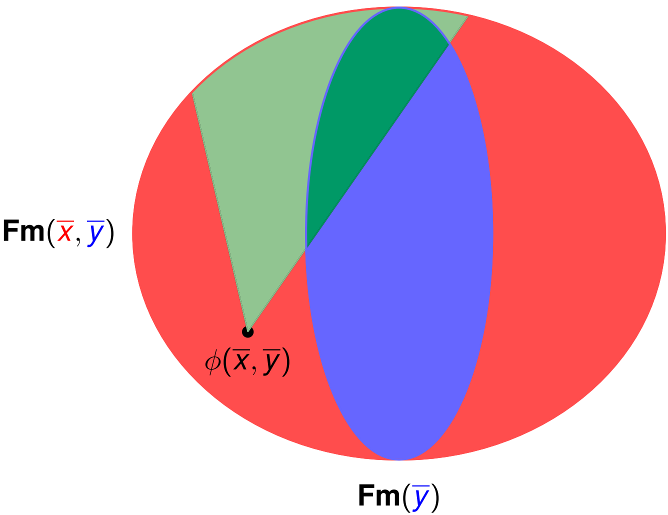

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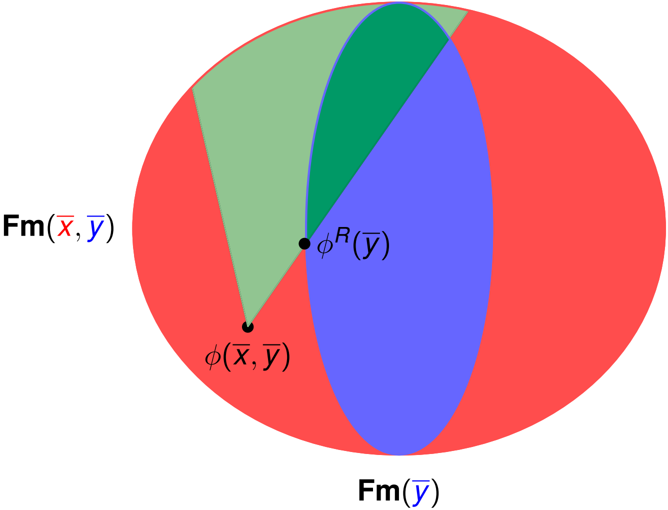

# Uniform interpolation in intermediate and modal logics

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- Ghilardi & Zawadowski: uniform interpolation holds for all intermediate logics admitting interpolation, and for **K**.
- Shavrukov: uniform interpolation holds for **GL**.
- Ghilardi & Zawadowski: uniform interpolation fails for **K4** and **S4**.

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- **Main question:** When does a variety of algebras admit uniform interpolation?
- In general, uniform interpolation is a property of **maps between compact congruences.**
- This observation leads to algebraic characterizations for existence of left and right uniform interpolants,
- and insight into the structure of the category of algebras.
- We apply the characterizations to certain varieties for substructural logics.

# The congruence lattice

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We denote by KCon(**A**) the join-semilattice of **compact congruences**.

# Lifting homomorphisms

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## Fact

*The pair $(f^*, f^{-1})$ is an* **adjunction**, *i.e., for any $\theta \in \mathrm{Con}(\mathbf{A}), \psi \in \mathrm{Con}(\mathbf{B})$,*

$$f^*(\theta) \subseteq \psi \iff \theta \subseteq f^{-1}(\psi).$$

# Lifting homomorphisms

For any $\overline{x}$ and $\overline{y}$ finite, let $i : \mathbf{F}_{\mathcal{V}}(\overline{y}) \hookrightarrow \mathbf{F}_{\mathcal{V}}(\overline{x}, \overline{y})$ denote the natural inclusion map.

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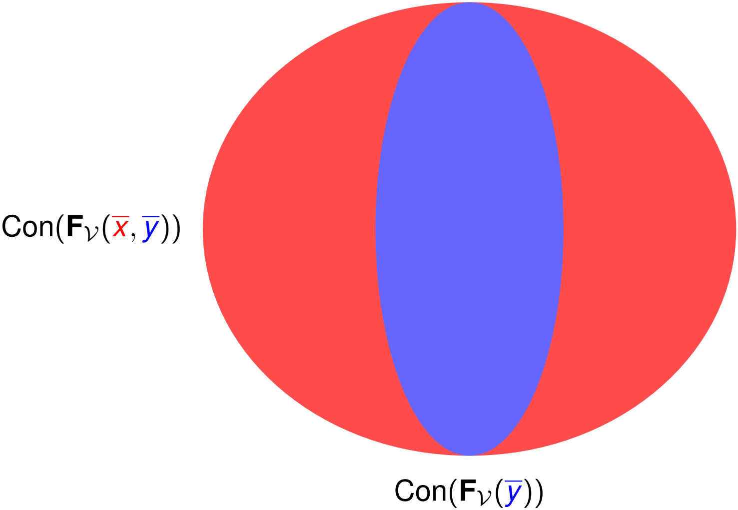

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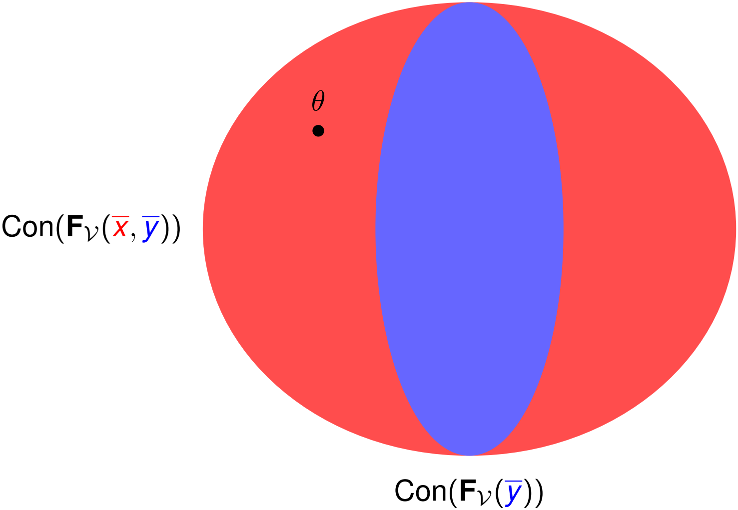

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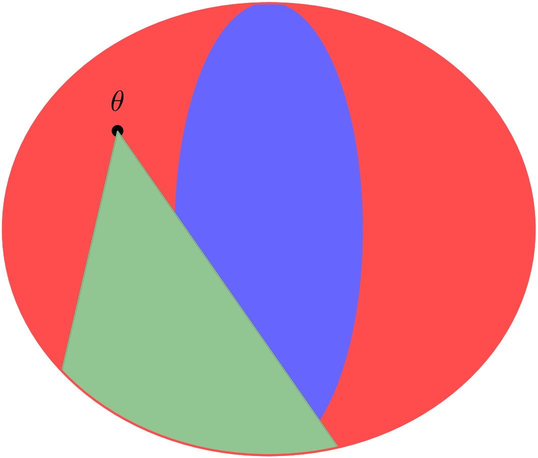


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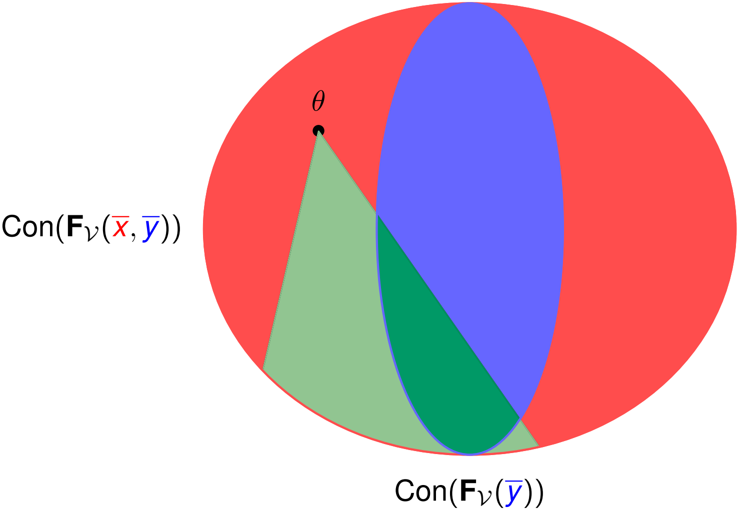

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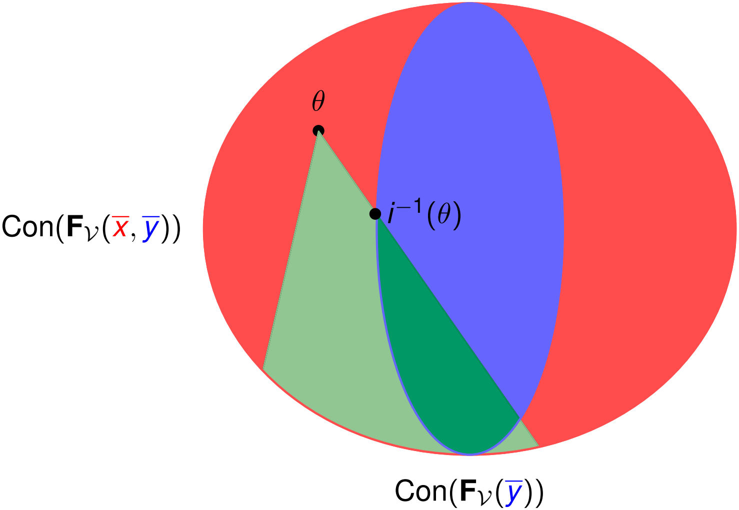

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**Remark (Heyting algebras)**

Any Heyting algebra $\mathbf{A}$ is dually isomorphic to $\mathrm{KCon}(\mathbf{A})$, so in this case the existence of a right adjoint to the compact lifting of $f$ is the same as the existence of a left adjoint to $f$.

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**Fact**

*Let* $\mathbf{A}$ *be an algebra in a variety* $\mathcal{V}$. *Then:*

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*Moreover, if* $\mathbf{A}$ *is finitely presented, then*

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2. KCon(**A**) *is dually isomorphic to the lattice of regular subobjects of* **A** *in the opposite category of finitely presented* $\mathcal{V}$*-algebras.*

Modulo these isomorphisms, the lifted adjunction $(f^*, f^{-1})$ is the adjunction $(\exists_f, f^*)$ between regular subobject lattices, used in categorical logic.

# Deductive interpolation

**Definition**

A variety $\mathcal{V}$ has deductive interpolation if, and only if, for any set of equations $\Sigma(\overline{x}, \overline{y})$ and equation $\epsilon(\overline{y}, \overline{z})$ such that $\Sigma \models_{\mathcal{V}} \epsilon$, there exists $\Delta(\overline{y})$ such that $\Sigma \models_{\mathcal{V}} \Delta$ and $\Delta \models_{\mathcal{V}} \epsilon$.

# Deductive vs. uniform interpolation

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## Definition

A variety $\mathcal{V}$ has right uniform deductive interpolation if, and only if, for any finite set of equations $\Sigma(\overline{x}, \overline{y})$, there exists a finite set of equations $\Pi(\overline{y})$ such that for any equation $\epsilon(\overline{y}, \overline{z})$,

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1. *$\mathcal{V}$ has right uniform deductive interpolation;*
2. *$\mathcal{V}$ has deductive interpolation, and for finite $\overline{x}$, $\overline{y}$, the compact lifting of $\mathbf{F}_{\mathcal{V}}(\overline{y}) \hookrightarrow \mathbf{F}_{\mathcal{V}}(\overline{x}, \overline{y})$ has a right adjoint;*

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3. *$\mathcal{V}$ has deductive interpolation, and the compact lifting of* **any** *homomorphism between finitely presented algebras in $\mathcal{V}$ has a right adjoint.*

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Note: The condition that $\mathrm{KCon}(\mathbf{F}_{\mathcal{V}}(\omega))$ is a Brouwerian join-semilattice is equivalent to $\mathcal{V}$ having equationally definable principal congruences (Köhler & Pigozzi 1980).

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# Examples, right case

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The following varieties have right uniform interpolation:

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However, the varieties of groups and of **S4**-algebras do not have right uniform interpolation.

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For example,

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is a consequence of both $\top \approx x$ and $\top \approx y$, i.e.,

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but there is no $\Delta(x, y)$ satisfying

$$\Delta \models_{\mathcal{ISL}} \Sigma, \quad \{\top \approx x\} \models_{\mathcal{ISL}} \Delta, \quad \text{and} \quad \{\top \approx y\} \models_{\mathcal{ISL}} \Delta.$$

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- Can we weaken these conditions to cover other classes of algebras, e.g., quasi-varieties, universal classes?

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