INTERPOLATION IN BROUWER LOGICS DETERMINED BY k-BRANCHING NETS OF CLUSTERS

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Brouwerian logic $\mathbf{KTB}$

Axioms CL and

$$K := \Box(p \to q) \to (\Box p \to \Box q)$$
$$T := \Box p \to p$$
$$B := p \to \Box\Diamond p$$

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$$alt_3 := \Box p \vee \Box(p \to q) \vee \Box((p \wedge q) \to r) \vee \Box((p \wedge q \wedge r) \to s)$$

Logic $\mathbf{KTB.alt_3}$ is complete with respect to the class of reflexive and symmetric Kripke frames being chains of points.

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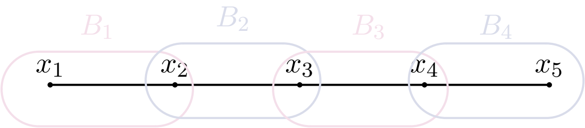

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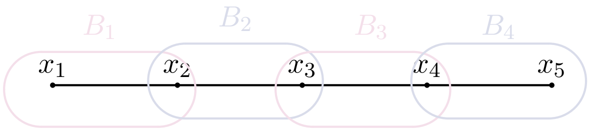

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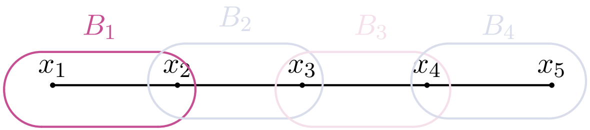

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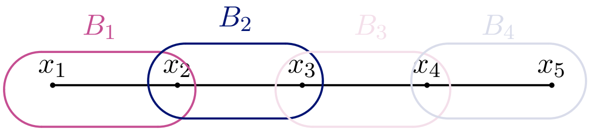

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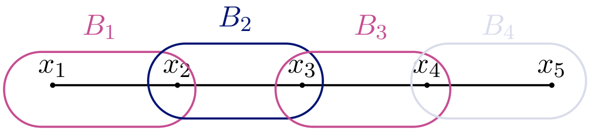

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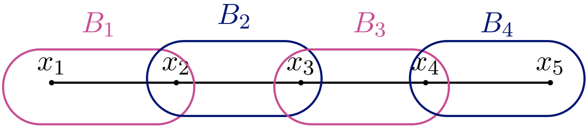

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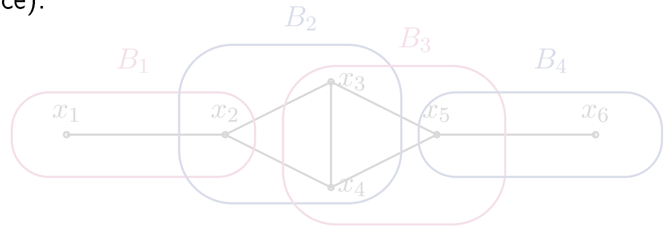

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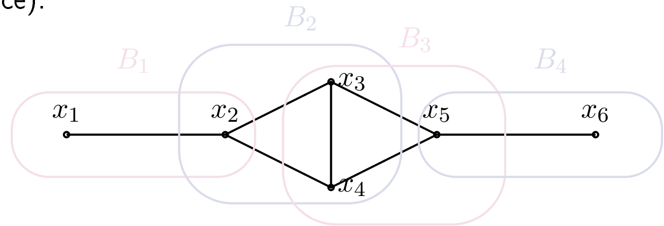

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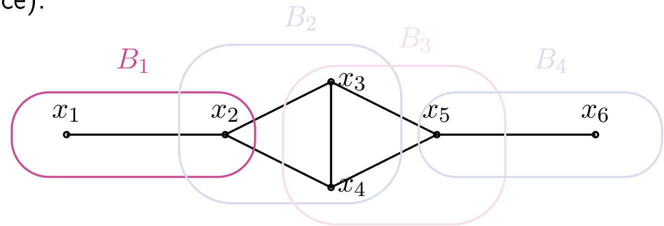

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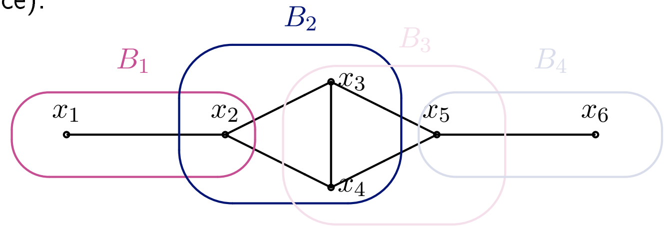

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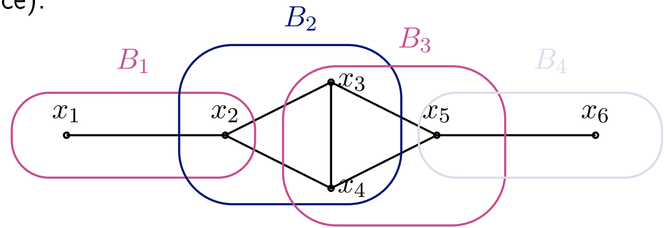

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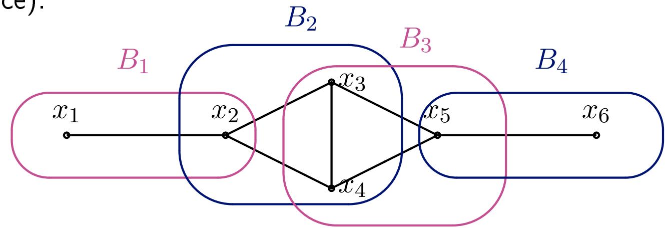

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The logic $\mathbf{KTB.3'}$ has the finite model property (f.m.p).

The class of reflexive and symmetric frames with linearly ordered blocks of tolerance is denoted by $\mathcal{LOB}$.

Theorem

Let $L \in NEXT(\mathbf{KTB.3'})$. Then L is Kripke complete with respect to the class of frames from $\mathcal{LOB}$ and has f.m.p.

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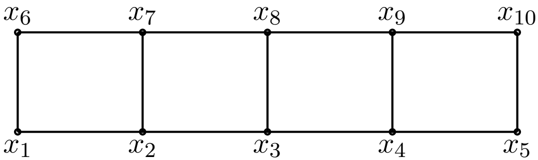

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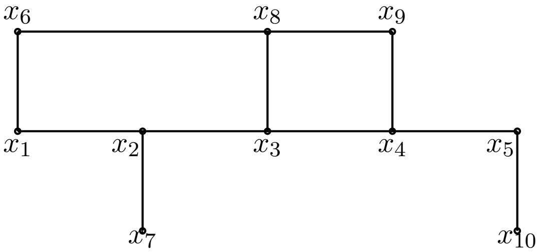

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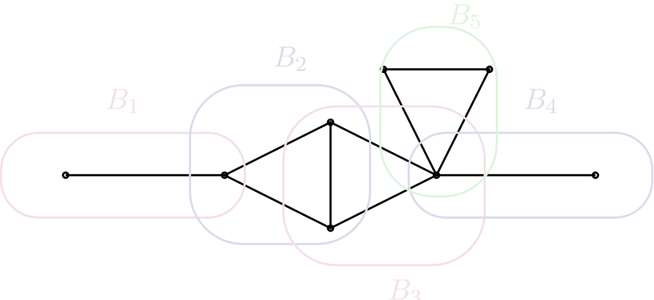

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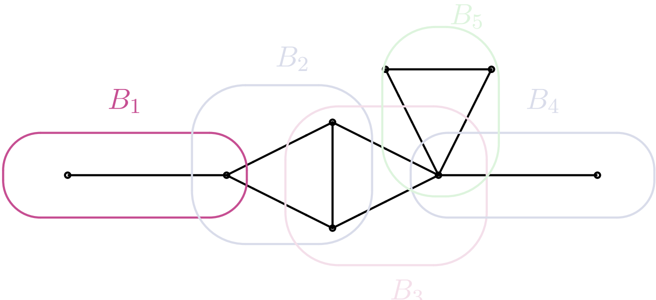

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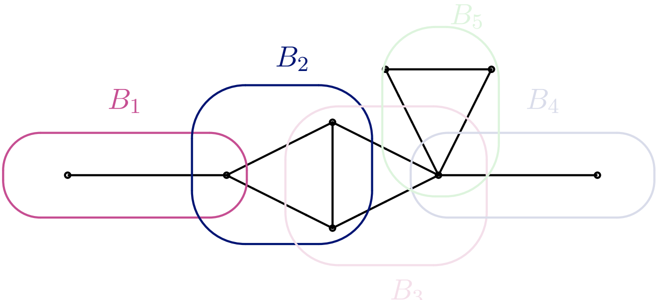

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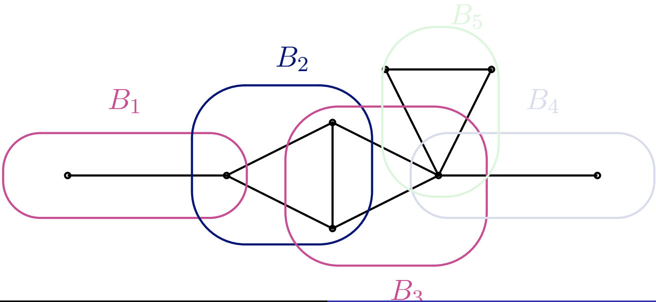

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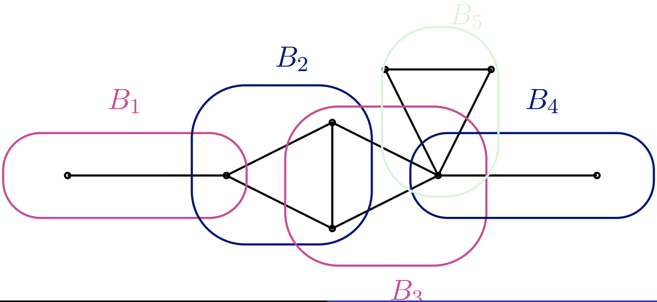

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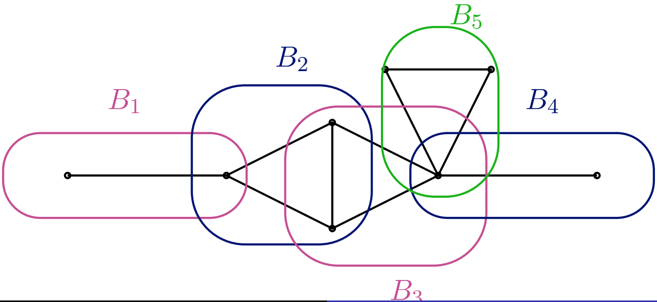

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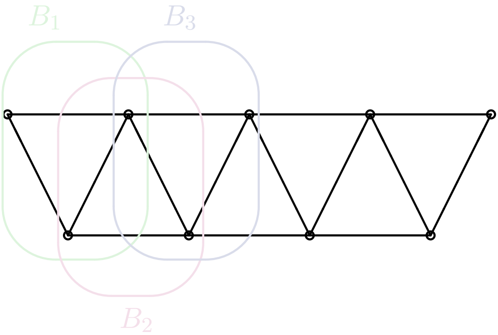

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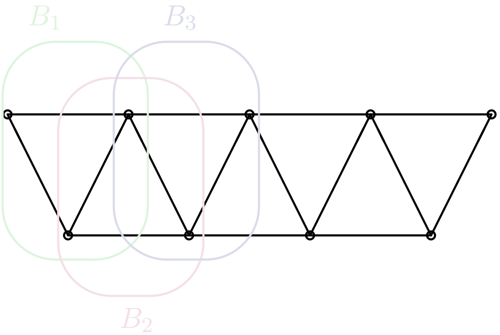

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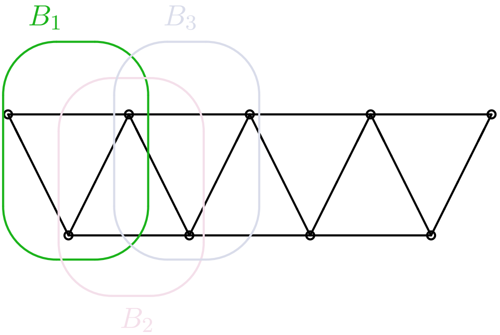

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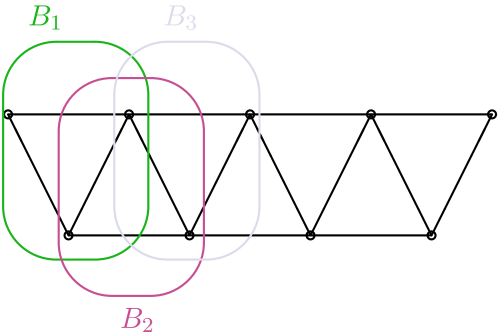

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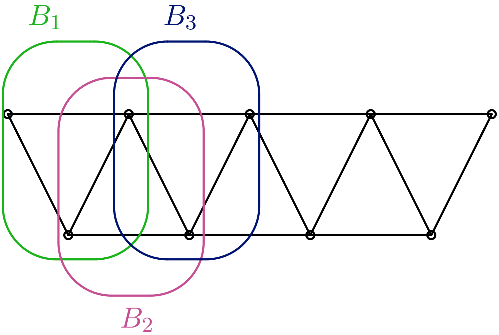

From $\mathbf{KTB.alt_n}$ to $\mathbf{KTB.n'}$

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- A logic L has the Craig interpolation property (CIP) if for every implication $\alpha \to \beta$ in L, there exists a formula γ such that

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The Brouwer logic $\mathbf{KTB}$ *have (CIP).*

Proof.(?) The method of construction of inseparable tableaux should work.

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How many normal extensions of $\mathbf{KTB.alt_3}$ and $\mathbf{KTB.3'}$ have (CIP) (or IDP)?

Theorem

If L has only one Post-complete extension and is Halldén-incomplete, then interpolation fails in L. [Schumm, 1986]

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Homogenous frames

Lemma

[van Benthem and Humberstone, 1983]
If a modal logic L is determined by one Kripke frame, which is homogeneous, then L is Halldén complete.

Lemma

[ZK and Y.Miyazaki 2013]
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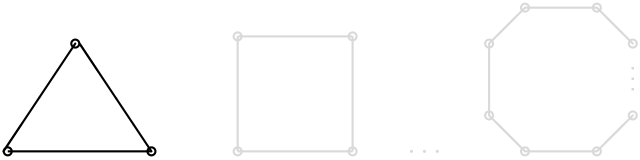
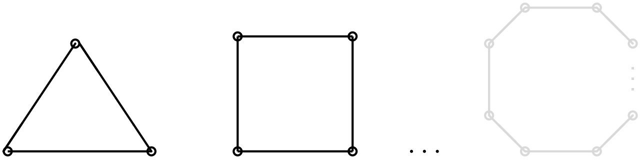


Two trivial circular frames:



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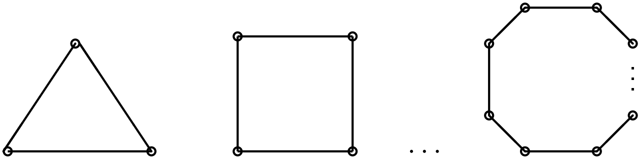
Finite homogeneous $KTB.alt_3$-frames - circular frames



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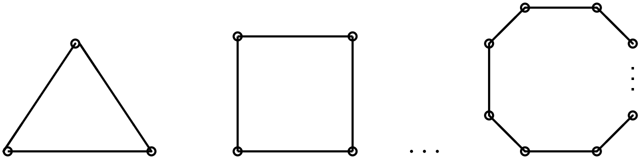

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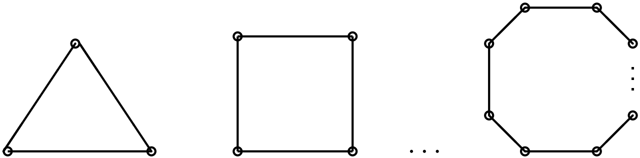

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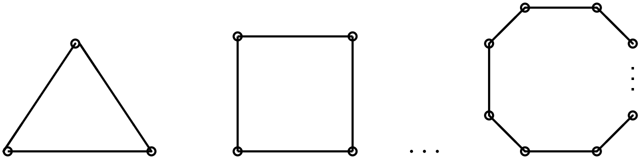

Two trivial circular frames:

Corollary

All tabular and Halldén complete logics in $NEXT(\mathbf{KTB.alt_3})$ *are determined by the circular frames:* $\mathfrak{C}_n$, $n \in \mathbb{N}$.

Question

Which logics $L(\mathfrak{C}_n)$, $n \in \mathbb{N}$ *have (IPD) or (CIP)?*

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Amalgamation for Kripke frames

Amalgamation property for frames (APK)
For any $\mathfrak{F}_0$, $\mathfrak{F}_1$ and $\mathfrak{F}_2$ in class K and for any p-morphism $f_1 : \mathfrak{F}_1 \to \mathfrak{F}_0$ and $f_2 : \mathfrak{F}_2 \to \mathfrak{F}_0$ there exist $\mathfrak{F}$ in K and p-morphisms $g_1 : \mathfrak{F} \to \mathfrak{F}_1$ and $g_2 : \mathfrak{F} \to \mathfrak{F}_2$ such that $f_1 \circ g_1 = f_2 \circ g_2$.

Superamalgamation property requires an additional condition (SAPK):

$$\forall_{x\in\mathfrak{F}_1}\forall_{y\in\mathfrak{F}_2}[f_1(x) = f_2(y) \;\Rightarrow\; \exists_{z\in\mathfrak{F}} g_1(z) = x \;\wedge\; g_2(z) = y].$$

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Theorem

There are only two tabular logics with (CIP) in $NEXT(\mathbf{KTB.alt_3})$. They are $L(\circ)$ and $L(\circ\!\!-\!\!\circ)$.

Proof. By superamalgamation property for frames.

Theorem

The logic $L(\mathfrak{C}_4)$ has (IPD) and do not has (CIP). It is the only logics among $L(\mathfrak{C}_n)$, $n \geq 3$ and n is finite.

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Interpolation in $NEXT(\mathbf{KTB.3'})$

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There are only three logics with (CIP) in $NEXT(\mathbf{KTB.3'})$. They are $L(\circ)$ and $L(\circ\!\!-\!\!\!-\!\!\circ)$ and $\mathbf{S5}$. Other logic with (IPD) and without (CIP) is the logic determined by four element chain of clusters.

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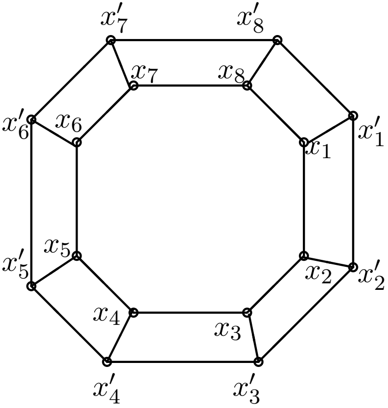
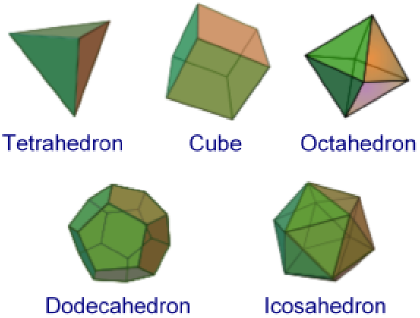
Finite, homogenous $KTB.alt_4$- frames



The diagram of reflexive, symmetric double circular frame $\mathfrak{DC}_{16}$

Finite, homogenous $KTB.alt_n$-frames, $n \geq 4$ - Platonic solids



Picture from wikipedia

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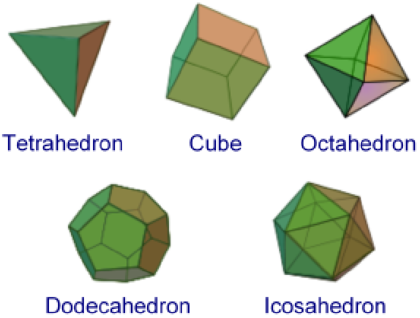

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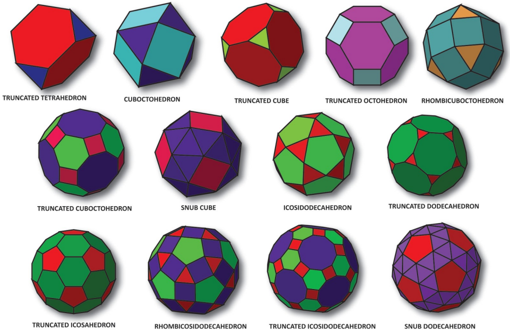

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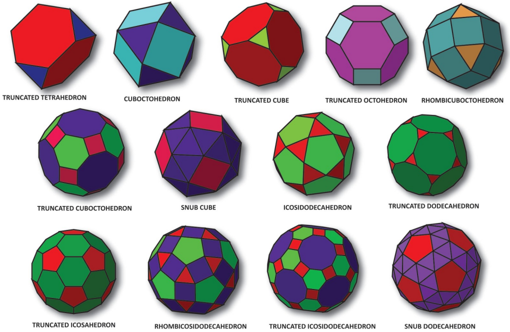

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Logics with (IPD) in $NEXT(KTB.alt_4)$

Theorem

The logic determined by the frame in a shape of cube has (IPD) and do not has (CIP).

Proof. By amalgamation property for frames.

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The logic determined by the frame in a shape of 16-element Boolean algebra has (IPD) and do not has (CIP).

Proof. By amalgamation property for frames.

And so on....

Logics with (IPD) in $NEXT(KTB.alt_5)$

Theorem

The logic determined by the frame in a shape of 16-element Boolean algebra has (IPD) and do not has (CIP).

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Future work

Description of the class of tabular logic with (IDP) in $NEXT(KTB.alt_4)$ and $NEXT(KTB.alt_5)$.

Proving that the Brouwer logic $\mathbf{KTB}$ have (CIP).

Proving that the logics $\mathbf{KTB.alt_n}$ and $\mathbf{KTB.n'}$, $n \geq 3$ do not have (CIP) and (IPD).

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J.F.A.K. van Benthem, I.I.Humberstone, *Halldén-completeness by Gluing of Kripke Frames*, NDJFL 24/4, (1983), 426–430.

A. Chagrow, M. Zakharyaschev, *On Halldén-completeness of intermediate and modal logics*, BSL 19/1 (1990), 21-–23.

J. Czelakowski, *Logical matrices and the amalgamation property*, SL 41, (4), (1981), 329–341.

S. Halldén, *On the semantic non-completeness of certain Lewis calculi*, JSL 16, (1951), 127–129.

Z. Kostrzycka, *On interpolation and Halldén-completeness in* $NEXT(\mathbf{KTB})$, BSL 41/1-2, 2012, 23–32.

Z. Kostrzycka, *On linear Brouwerian logics*, Mathematical Logic Quarterly 60 (4-5): (2014), 304—313.

S. A. Kripke, *Semantical analysis of modal logic I.*, Zeitschr. f. math. Logik und Grundlagen d. Math., 9 (1963), 67–96.

Thank you for your attention.