Jónsson-style Canonicity for ALBA-Inequalities (Unified Corespondence I)

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TACL, 26 June 2015

Correspondence via Duality

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Correspondence theory arises semantically:

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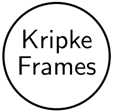

Correspondence via Duality

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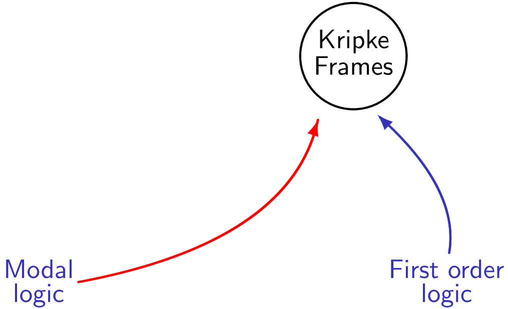

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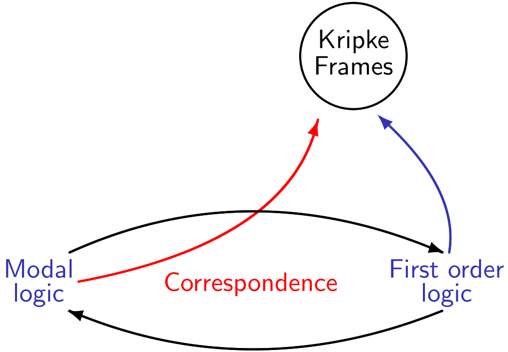
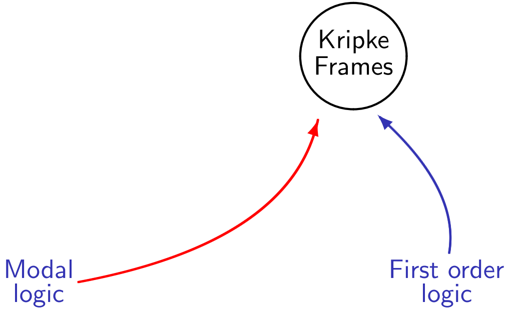


Correspondence via Duality

An asymmetry:

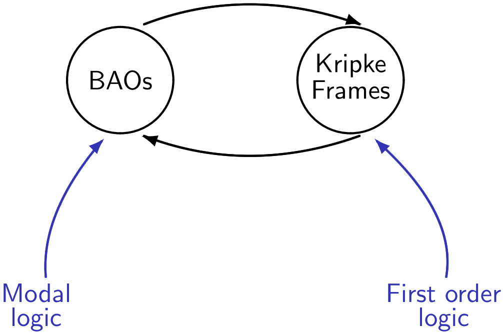


Non canonical interpretation

Canonical interpretation

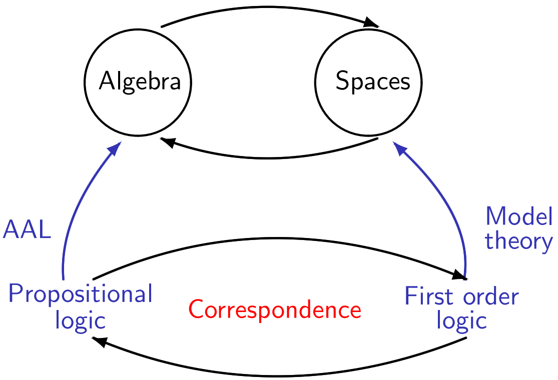
Correspondence via Duality

Symmetry re-established via duality:



Correspondence via Duality

Correspondence available not just for normal modal logic:



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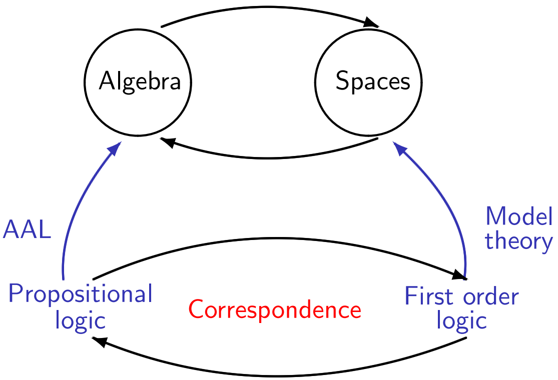

- specific correspondences as logical reflections of dualities
- dual characterizations as instances of Generalized Correspondence

A calculus mechanizing minimal valuation meta-arguments

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Transitivity:

$$\forall p[\Diamond\Diamond p \leq \Diamond p]$$

$$\text{iff} \quad \forall p \forall i \forall m[(i \leq \Diamond\Diamond p \ \&\ \Diamond p \leq m) \Rightarrow i \leq m]$$

A calculus mechanizing minimal valuation meta-arguments

Transitivity:

$$\forall p[\Diamond\Diamond p \leq \Diamond p]$$

$$\begin{array}{lll} & \text{iff} & \forall p \forall i \forall m[(i \leq \Diamond\Diamond p \ \& \ \Diamond p \leq m) \Rightarrow i \leq m] \\ \text{Ack} & \text{iff} & \forall p \forall i \forall m \forall j[(i \leq \Diamond\Diamond j \ \& \ j \leq p \ \& \ \Diamond p \leq m) \Rightarrow i \leq m] \end{array}$$

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A calculus mechanizing minimal valuation meta-arguments

Transitivity:

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	iff	$\forall i \forall j[i \leq \Diamond\Diamond j \Rightarrow i \leq \Diamond j]$
	iff	$\forall j[\Diamond\Diamond j \leq \Diamond j]$
	iff	$\forall w[R^{-1}[R^{-1}[w]] \subseteq R^{-1}[w]]$
	iff	$\forall w[R[R[w]] \subseteq R[w]]$.

Unified correspondence

Hybrid logics
[CR15]

DLE-logics
[CP12, CPS]

Substructural logics
[CP15]

Mu-calculi
[CFPS15, CGP14, CC15]

Display calculi
[GMPTZ]

Regular DLE-logics
Kripke frames with
impossible worlds
[PSZ15a]

Jónsson-style vs
Sambin-style canonicity
[PSZ15b]

Finite lattices and
monotone ML
[FPS15]

Canonicity via
pseudo-correspondence
[CPSZ]

Sahlqvist Correspondence & Canonicity

Sahlqvist theory

sufficient **syntactic** conditions on modal formulas:

- to have a first order correspondent;
- to be canonical.

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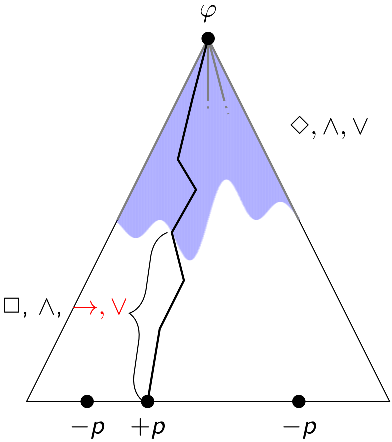

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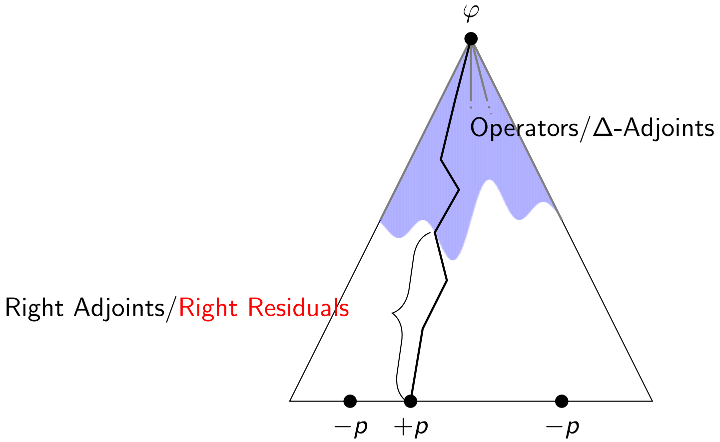

Two Approaches to Algebraic Canonicity

Canonicity: $\mathbb{A} \models \varphi \leq \psi \;\Rightarrow\; \mathbb{A}^\delta \models \varphi \leq \psi$

Jónsson-style

Via-Correspondence

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Via-Correspondence

Two Approaches to Algebraic Canonicity

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Via-Correspondence

$$\mathbb{G} \models \varphi \leq \psi \qquad\qquad \mathbb{F} \models \varphi \leq \psi$$

$$\Updownarrow$$

$$\mathbb{F} \models_{\mathbb{G}} \varphi \leq \psi \qquad\qquad \Updownarrow$$

$$\Updownarrow$$

$$\mathbb{F} \models_{\mathbb{G}} \mathsf{FO}(\varphi \leq \psi) \Leftrightarrow \mathbb{F} \models \mathsf{FO}(\varphi \leq \psi)$$

Two Approaches to Algebraic Canonicity

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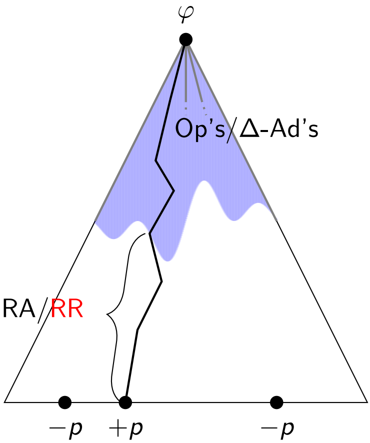
Via-Correspondence

Decompositional Strategy



$\mathbb{G} \models \varphi \leq \psi$

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$\mathbb{F} \models \varphi \leq \psi$

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Jónsson-style

$$\mathbb{A} \models \varphi \leq \psi$$

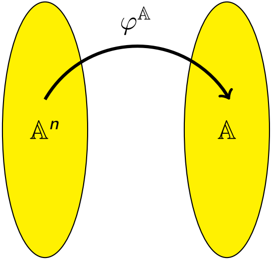
$$\Updownarrow$$

$$\varphi^{\mathbb{A}} \leq \psi^{\mathbb{A}}$$

$$\varphi^{\mathbb{A}^\delta} \qquad \leq \qquad \psi^{\mathbb{A}^\delta}$$

$$\Updownarrow$$

$$\mathbb{A}^\delta \models \varphi \leq \psi$$



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Canonicity: $\mathbb{A} \models \varphi \leq \psi \;\Rightarrow\; \mathbb{A}^{\delta} \models \varphi \leq \psi$

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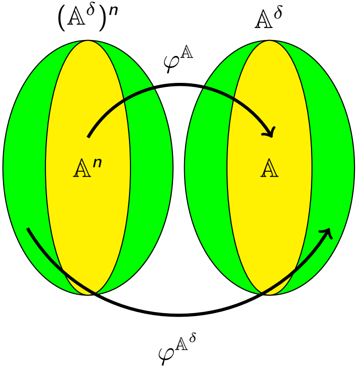
$$\Updownarrow$$

$$\varphi^{\mathbb{A}} \leq \psi^{\mathbb{A}}$$

$$\varphi^{\mathbb{A}^{\delta}} \qquad \leq \qquad \psi^{\mathbb{A}^{\delta}}$$

$$\Updownarrow$$

$$\mathbb{A}^{\delta} \models \varphi \leq \psi$$



Two Approaches to Algebraic Canonicity

Canonicity: $\mathbb{A} \models \varphi \leq \psi \;\Rightarrow\; \mathbb{A}^\delta \models \varphi \leq \psi$

Jónsson-style

$$\mathbb{A} \models \varphi \leq \psi$$

$$\Updownarrow$$

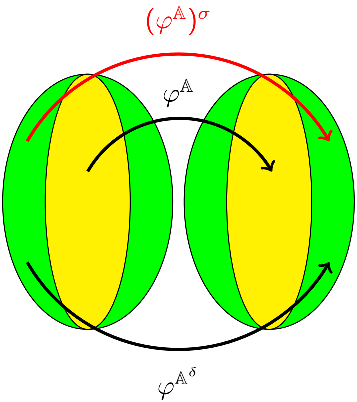
$$\varphi^{\mathbb{A}} \leq \psi^{\mathbb{A}}$$

$$\Downarrow$$

$$\varphi^{\mathbb{A}^\delta} \leq (\varphi^{\mathbb{A}})^\sigma \leq (\psi^{\mathbb{A}})^\sigma \leq \psi^{\mathbb{A}^\delta}$$

$$\Updownarrow$$

$$\mathbb{A}^\delta \models \varphi \leq \psi$$



Canonicity: $\mathbb{A} \models \varphi \leq \psi \;\Rightarrow\; \mathbb{A}^\delta \models \varphi \leq \psi$

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$$\mathbb{A} \models \varphi \leq \psi$$

$$\Updownarrow$$

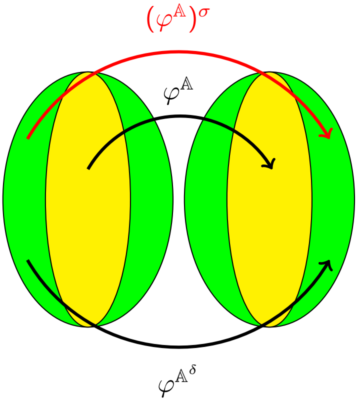
$$\varphi^{\mathbb{A}} \leq \psi^{\mathbb{A}}$$

$$\Downarrow$$

$$\varphi^{\mathbb{A}^\delta} \leq (\varphi^{\mathbb{A}})^\sigma \leq (\psi^{\mathbb{A}})^\sigma \leq \psi^{\mathbb{A}^\delta}$$

σ-expanding $\Updownarrow$ σ-contracting

$$\mathbb{A}^\delta \models \varphi \leq \psi$$



Two Approaches to Algebraic Canonicity

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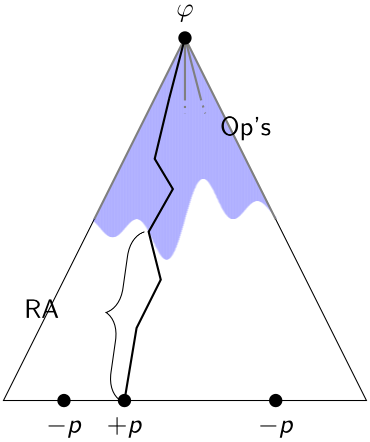
σ-expanding $\Updownarrow$ σ-contracting

$$\mathbb{A}^\delta \models \varphi \leq \psi$$

Compositional Strategy



Same conditions, two strategies

Jónsson-style

$\mathbb{A} \models \varphi \leq \psi$

$\Updownarrow$

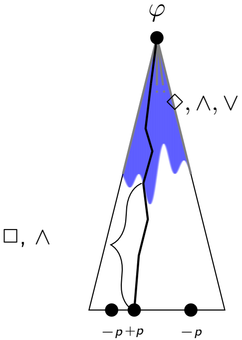
$\varphi^{\mathbb{A}} \leq \psi^{\mathbb{A}}$

$\Downarrow$

$\varphi^{\mathbb{A}^{\delta}} \leq (\varphi^{\mathbb{A}})^{\sigma} \leq (\psi^{\mathbb{A}})^{\sigma} \leq \psi^{\mathbb{A}^{\delta}}$

σ-exp. $\Updownarrow$ σ-contr.

$\mathbb{A}^{\delta} \models \varphi \leq \psi$



Via-Correspondence

$\mathbb{G} \models \varphi \leq \psi$		$\mathbb{F} \models \varphi \leq \psi$
$\Updownarrow$		
$\mathbb{F} \models_{\mathbb{G}} \varphi \leq \psi$		$\Updownarrow$
$\Updownarrow$		
$\mathbb{F} \models_{\mathbb{G}} \mathrm{FO}(\varphi \leq \psi)$	$\Leftrightarrow$	$\mathbb{F} \models \mathrm{FO}(\varphi \leq \psi)$

Same conditions, two strategies

Jónsson-style

$\mathbb{A} \models \varphi \leq \psi$

$\Updownarrow$

$\varphi^{\mathbb{A}} \leq \psi^{\mathbb{A}}$

$\Downarrow$

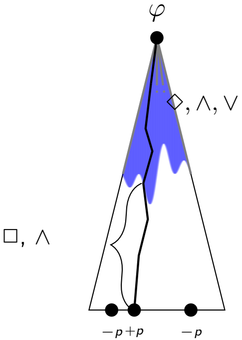
$\varphi^{\mathbb{A}^\delta} \leq (\varphi^{\mathbb{A}})^\sigma \leq (\psi^{\mathbb{A}})^\sigma \leq \psi^{\mathbb{A}^\delta}$

σ-exp. $\Updownarrow$ σ-contr.

$\mathbb{A}^\delta \models \varphi \leq \psi$

Success

Sahlqvist



Via-Correspondence

$\mathbb{G} \models \varphi \leq \psi$ $\qquad$ $\mathbb{F} \models \varphi \leq \psi$

$\Updownarrow$

$\mathbb{F} \models_{\mathbb{G}} \varphi \leq \psi$ $\qquad$ $\Updownarrow$

$\Updownarrow$

$\mathbb{F} \models_{\mathbb{G}} \mathrm{FO}(\varphi \leq \psi) \Leftrightarrow \mathbb{F} \models \mathrm{FO}(\varphi \leq \psi)$

Success

Same conditions, two strategies

Jónsson-style

$$\mathbb{A} \models \varphi \leq \psi$$

$$\Updownarrow$$

$$\varphi^{\mathbb{A}} \leq \psi^{\mathbb{A}}$$

$$\Downarrow$$

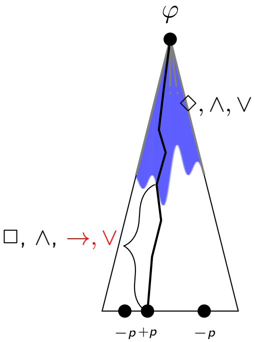
$$\varphi^{\mathbb{A}^\delta} \leq (\varphi^{\mathbb{A}})^\sigma \leq (\psi^{\mathbb{A}})^\sigma \leq \psi^{\mathbb{A}^\delta}$$

σ-exp. $\Updownarrow$ σ-contr.

$$\mathbb{A}^\delta \models \varphi \leq \psi$$

???

Inductive



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$$\mathbb{G} \models \varphi \leq \psi \qquad\qquad \mathbb{F} \models \varphi \leq \psi$$

$$\Updownarrow$$

$$\mathbb{F} \models_{\mathbb{G}} \varphi \leq \psi \qquad\qquad \Updownarrow$$

$$\Updownarrow$$

$$\mathbb{F} \models_{\mathbb{G}} \mathrm{FO}(\varphi \leq \psi) \iff \mathbb{F} \models \mathrm{FO}(\varphi \leq \psi)$$

Success

Same conditions, two strategies

Jónsson-style

$\mathbb{A} \vDash \varphi \leq \psi$

$\Updownarrow$

$\varphi^{\mathbb{A}} \leq \psi^{\mathbb{A}}$

$\Downarrow$

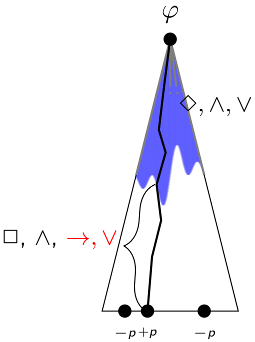
$\varphi^{\mathbb{A}^\delta} \leq (\varphi^{\mathbb{A}})^\sigma \leq (\psi^{\mathbb{A}})^\sigma \leq \psi^{\mathbb{A}^\delta}$

σ-exp. $\Updownarrow$ σ-contr.

$\mathbb{A}^\delta \vDash \varphi \leq \psi$

???

Inductive



Via-Correspondence

$\mathbb{G} \vDash \varphi \leq \psi$		$\mathbb{F} \vDash \varphi \leq \psi$
$\Updownarrow$		
$\mathbb{F} \vDash_{\mathbb{G}} \varphi \leq \psi$		$\Updownarrow$
$\Updownarrow$		
$\mathbb{F} \vDash_{\mathbb{G}} \mathrm{FO}(\varphi \leq \psi)$	$\Leftrightarrow$	$\mathbb{F} \vDash \mathrm{FO}(\varphi \leq \psi)$

Success

- Conceptual Aim: Understanding how they compare.
- Technical Aim: Jónsson-style canonicity for Inductive inequalities.

Combining the two strategies

Let $\varphi \leq \psi$ be inductive.

$$\mathbb{A} \models \varphi \leq \psi \qquad \overset{?}{\Rightarrow} \qquad \mathbb{A}^\delta \models \varphi \leq \psi$$

Combining the two strategies

Let $\varphi \leq \psi$ be inductive.

$$\mathbb{A} \models \varphi \leq \psi$$

$$\Updownarrow$$

$$\mathbb{A}^\delta \models_{\mathbb{A}} \varphi \leq \psi$$

$$\Updownarrow$$

$$\mathbb{A}^\delta \models_{\mathbb{A}} \mathsf{FO}(\varphi \leq \psi) \quad \Leftrightarrow \quad \mathbb{A}^\delta \models \mathsf{FO}(\varphi \leq \psi)$$

$$\Updownarrow$$

$$\mathbb{A}^\delta \models \varphi \leq \psi$$

Combining the two strategies

Let $\varphi \leq \psi$ be inductive.

$$\begin{array}{ccc}
\mathbb{A} \models \varphi \leq \psi & & \mathbb{A}^\delta \models \varphi \leq \psi \\
\Updownarrow & & \\
\mathbb{A}^\delta \models_{\mathbb{A}} \varphi \leq \psi & & \Updownarrow \\
\Updownarrow & ? & \\
\mathbb{A}^\delta \models_{\mathbb{A}} \mathsf{PREFO}(\varphi \leq \psi) & \Rightarrow & \mathbb{A}^\delta \models \mathsf{PREFO}(\varphi \leq \psi) \\
\Updownarrow & & \Updownarrow \\
\mathbb{A}^\delta \models_{\mathbb{A}} \mathsf{FO}(\varphi \leq \psi) & \Leftrightarrow & \mathbb{A}^\delta \models \mathsf{FO}(\varphi \leq \psi)
\end{array}$$

Combining the two strategies

Let $\varphi \leq \psi$ be inductive.

$\mathbb{A} \models \varphi \leq \psi$		$\mathbb{A}^\delta \models \varphi \leq \psi$
$\Updownarrow$		
$\mathbb{A}^\delta \models_{\mathbb{A}} \varphi \leq \psi$		$\Updownarrow$
$\Updownarrow$	?	
$\mathbb{A}^\delta \models_{\mathbb{A}}$ PREFO$(\varphi \leq \psi)$	$\Rightarrow$	$\mathbb{A}^\delta \models$ PREFO$(\varphi \leq \psi)$

Combining the two strategies

Let $\varphi \leq \psi$ be inductive.

$$\begin{array}{ccc}
\mathbb{A} \models \varphi \leq \psi & & \mathbb{A}^\delta \models \varphi \leq \psi \\
\Updownarrow & & \\
\mathbb{A}^\delta \models_{\mathbb{A}} \varphi \leq \psi & & \Updownarrow \\
\Updownarrow & & \\
\mathbb{A}^\delta \models_{\mathbb{A}} \alpha \leq \beta & & \mathbb{A}^\delta \models \alpha \leq \beta \\
\Updownarrow & ? & \Updownarrow \\
\alpha^{\mathbb{A}} \leq \beta^{\mathbb{A}} & \Rightarrow & \alpha^{\mathbb{A}^\delta} \leq \beta^{\mathbb{A}^\delta}
\end{array}$$

Combining the two strategies

Let $\varphi \leq \psi$ be inductive.

$$\mathbb{A} \vDash \varphi \leq \psi$$
$$\Updownarrow$$
$$\mathbb{A}^\delta \vDash_{\mathbb{A}} \varphi \leq \psi$$
$$\Updownarrow$$
$$\mathbb{A}^\delta \vDash_{\mathbb{A}} \alpha \leq \beta$$
$$\Updownarrow$$
$$\alpha^{\mathbb{A}} \leq \beta^{\mathbb{A}}$$

generalized Sahlqvist

(in expanded language)

$$\mathbb{A}^\delta \vDash \varphi \leq \psi$$
$$\Updownarrow$$
$$\mathbb{A}^\delta \vDash \alpha \leq \beta$$
$$\Updownarrow$$
$$\alpha^{\mathbb{A}^\delta} \leq \beta^{\mathbb{A}^\delta}$$

Combining the two strategies

Let $\varphi \leq \psi$ be inductive.

$$\mathbb{A} \models \varphi \leq \psi \quad\quad \mathbb{A}^\delta \models \varphi \leq \psi$$

$$\Updownarrow$$

$$\mathbb{A}^\delta \models_{\mathbb{A}} \varphi \leq \psi \quad\quad \Updownarrow$$

$$\Updownarrow$$

$$\mathbb{A}^\delta \models_{\mathbb{A}} \alpha \leq \beta \quad\quad \mathbb{A}^\delta \models \alpha \leq \beta$$

$$\Updownarrow \quad\quad \Updownarrow$$

$$\alpha^{\mathbb{A}} \leq \beta^{\mathbb{A}} \quad \Rightarrow \quad \alpha^{\mathbb{A}^\delta} \leq (\alpha^{\mathbb{A}})^\sigma \leq (\beta^{\mathbb{A}})^\sigma \leq \beta^{\mathbb{A}^\delta}$$

generalized Sahlqvist (in expanded language)

generalized σ-expanding   generalized σ-contracting (using generalized canonical extension)

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[Palmigiano Sourabh Zhao/b] Jónsson-style canonicity for ALBA inequalities, *JLC*, 2015.