Sahlqvist Theory for Hybrid Logics (Unified Correspondence IV)

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Unified correspondence

Hybrid logics
[CR15]

DLE-logics
[CP12, CPS]

Substructural logics
[CP15]

Mu-calculi
[CFPS15, CGP14, CC15]

Display calculi
[GMPTZ]

Regular DLE-logics
Kripke frames with
impossible worlds
[PSZ15a]

Jónsson-style vs
Sambin-style canonicity
[PSZ15b]

Finite lattices and
monotone ML
[FPS15]

Canonicity via
pseudo-correspondence
[CPSZ]

Languages

Fix countably infinite disjoint sets PROP and NOM of propositional variables ($p, q, r, \ldots$) and nominals ($\mathbf{i}, \mathbf{j}, \mathbf{k}, \ldots$), respectively. Then $\mathcal{H}(@)$ is defined as follows:

$$\varphi ::= \bot \mid p \mid \mathbf{i} \mid \neg\varphi \mid \varphi \wedge \psi \mid \Diamond\varphi \mid @_{\mathbf{j}}\varphi,$$

with $p \in$ PROP and $\mathbf{i} \in$ NOM.

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with $p \in \text{PROP}$ and $\mathbf{i} \in \text{NOM}$.
The expanded language $\mathcal{H}^{+}(@)$ is defined as follows:

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with $p \in$ PROP and $\mathbf{i} \in$ NOM.
Let $\varphi, \psi, \varphi_1, \ldots, \varphi_n, \psi_1, \ldots, \psi_n \in \mathcal{H}^+(@)$. Then:

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Let $\varphi, \psi, \varphi_1, \ldots, \varphi_n, \psi_1, \ldots, \psi_n \in \mathcal{H}^{+}(@)$. Then:

- an inequality is an expression of the form $\varphi \leq \psi$;
- a quasi-inequality is an expression of the form $\varphi_1 \leq \psi_1 \ \& \ \cdots \& \ \varphi_n \leq \psi_n \Rightarrow \varphi \leq \psi$.

Models

$\mathcal{M} = (W, R, V)$ with

- $W \neq \emptyset$
- $R \subseteq W \times W$
- $V : \text{PROP} \to \mathcal{P}(W)$
- $V : \text{NOM} \to \{\{w\} \mid w \in W\}$

Relational semantics

- $\mathcal{M}, w \models p$ iff $w \in V(p)$;
- $\mathcal{M}, w \models \mathbf{i}$ iff $V(\mathbf{i}) = \{w\}$;
- $\mathcal{M}, w \models \neg\varphi$ iff $\mathcal{M}, w \not\models \varphi$;
- $\mathcal{M}, w \models \varphi \wedge \psi$ iff $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$;
- $\mathcal{M}, w \models \Diamond\varphi$ iff $\mathcal{M}, v \models \varphi$ for some $v \in W$ with wRv;
- $\mathcal{M}, w \models @_{\mathbf{i}}\varphi$ iff $\mathcal{M}, v \models \varphi$ where $V(\mathbf{i}) = \{v\}$;
- $\mathcal{M}, w \models \Diamond^{-1}\varphi$ iff $\mathcal{M}, v \models \varphi$ for some $v \in W$ with vRw;
- $\mathcal{M}, w \models \mathrm{E}\varphi$ iff $\mathcal{M}, v \models \varphi$ for some $v \in W$.

Hybrid algebras

Definition

A hybrid algebra is a pair $\mathfrak{A} = (\mathbf{A}, X_A)$, where $\mathbf{A} = (A, \wedge, \vee, \neg, \bot, \top, \Diamond)$ such that $(A, \wedge, \vee, \neg, \bot, \top, \Diamond)$ is a BAO containing at least one atom and $\varnothing \neq X_A \subseteq At\mathbf{A}$.

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Definition

Any hybrid algebra can be turned into a hybrid algebra for $\mathcal{H}(@)$ by adding a binary operator @ whose first coordinate ranges over X_A and the second coordinate over all elements of the algebra, defined by

$$@_x a = \begin{cases} \top & \text{if } x \leq a \\ \bot & \text{otherwise} \end{cases}$$

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A permeated hybrid algebra is a hybrid algebra $\mathfrak{A} = (A, X_A)$ satisfying the following additional conditions:

1. for each $\bot \neq a \in A$, there is an atom $x \in X_A$ such that $x \leq a$, and
2. for all $x \in X_A$ and $a \in A$, if $x \leq \Diamond a$, then there exists a $y \in X_A$ such that $y \leq a$ and $x \leq \Diamond y$.

Algebraic semantics

Definition

An assignment on a hybrid algebra $(\mathbf{A}, X_A)$ is a map v associating an element of A with each propositional variable in PROP and an atom of X_A with each nominal in NOM. Given an assignment v, we calculate the meaning $\tilde{v}(t)$ of a term t as follows:

1. $v(\bot) = \bot$;
2. $\tilde{v}(p) = v(p)$;
3. $\tilde{v}(\mathbf{i}) = v(\mathbf{i})$;
4. $\tilde{v}(\neg\varphi) = \neg\tilde{v}(\varphi)$;
5. $\tilde{v}(\varphi \wedge \psi) = \tilde{v}(\varphi) \wedge \tilde{v}(\psi)$;
6. $\tilde{(}\Diamond\varphi) = \Diamond\tilde{v}(\varphi)$;
7. $\tilde{v}(@_{\mathbf{i}}\varphi) = @_{\tilde{v}(\mathbf{i})}\tilde{v}(\varphi)$;
8. $\tilde{v}(\mathrm{E}\varphi) = \mathrm{E}\tilde{v}(\varphi)$.

An inequality $\varphi \leq \psi$ is true in a hybrid algebra $\mathfrak{A}$ ($\mathfrak{A} \models \varphi \leq \psi$) if for all assignments v, $v(\varphi) \leq v(\psi)$.

A quasi-inequality $\varphi_1 \leq \psi_1 \ \& \ \cdots \& \ \varphi_n \leq \psi_n \Rightarrow \varphi \leq \psi$ is true in a hybrid algebra $\mathfrak{A}$ under assignment v, if $\varphi_i \leq \psi_i$ is *not* true in $\mathfrak{A}$ under v for some $1 \leq i \leq n$, or $\varphi \leq \psi$ is true in $\mathfrak{A}$ under v.

Admissible validity

Let $\mathfrak{A} = (\mathbf{A}, X_A)$ be a hybrid subalgebra of $\mathfrak{B} = (\mathbf{B}, X_B)$, i.e, $\mathbf{A}$ is a subalgebra of $\mathbf{B}$ and $X_A \subseteq X_B$. An admissible assignment in $\mathfrak{B}$ relative to $\mathfrak{A}$ is any assignment sending propositional variables into A and nominals into X_A.

We say that an inequality $\varphi \leq \psi$ is admissibly valid in $\mathfrak{B}$ relative to $\mathfrak{A}$, denoted $\mathfrak{B} \models_{\mathfrak{A}} \varphi \leq \psi$, if $\mathfrak{B}, v \models \varphi \leq \psi$ for every admissible assignment v relative to $\mathfrak{A}$.

Note that if $\varphi, \psi \in \mathcal{H}(@)$, then $\mathfrak{B} \models_{\mathfrak{A}} \varphi \leq \psi$ iff $\mathfrak{A} \models \varphi \leq \psi$.

Canonical extensions and MacNeille completions

Definition

The canonical extension of a hybrid algebra $\mathfrak{A} = (\mathbf{A}, X_A)$ is the hybrid algebra $\mathfrak{A}^\delta = (\mathbf{A}^\delta, X_{A^\delta})$ such that $\mathbf{A}^\delta$ is the canonical extension of $\mathbf{A}$ and $X_{A^\delta} = At\mathbf{A}^\delta$.

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Definition

We define the MacNeille completion of a hybrid algebra $\mathfrak{A} = (\mathbf{A}, X_A)$ to be the hybrid algebra $\mathfrak{A}^{dm} = (\mathbf{A}^{dm}, X_{A^{dm}})$ such that $\mathbf{A}^{dm}$ is the MacNeille completion of $\mathbf{A}$ and $X_{A^{dm}} = At\mathbf{A}^{dm}$.

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- If Σ is a set of pure formulas (formulas containing no propositional variables), then $\mathbf{H}^{+}(@) \oplus \Sigma$ is strongly complete with respect to its class of Kripke frames [G. Gargov and V. Goranko].
- The second and third result cannot be combined in general: the logic $\mathbf{H}^{+}(@) \oplus \{\Diamond\Box p \to \Box\Diamond p, \Diamond(\mathbf{i} \wedge \Box\mathbf{j}) \to \Box(\Diamond\mathbf{j} \to \mathbf{i})\}$ is incomplete [Ten Cate, Marx and Viana].

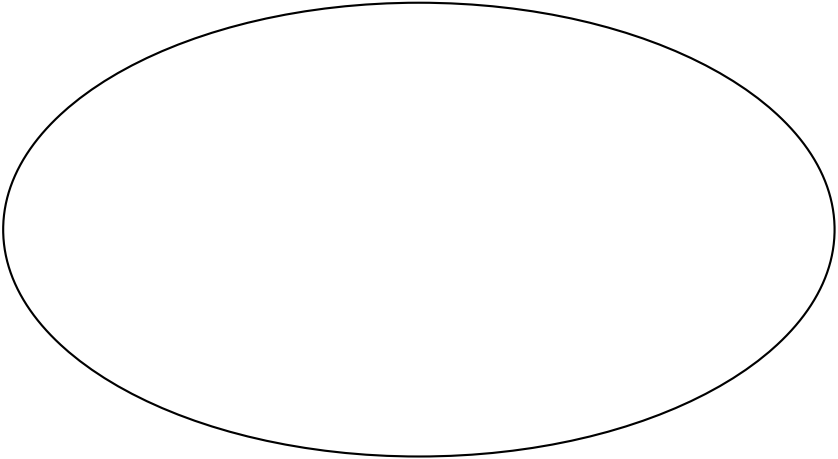
The logics $\mathbf{H}(@)$ and $\mathbf{H}^{+}(@)$: axioms

(*Taut*)	$\vdash \varphi$ for all propositional tautologies φ.
(*K*)	$\vdash \Box(p \to q) \to (\Box p \to \Box q)$
(*Dual*)	$\vdash \Diamond p \leftrightarrow \neg\Box\neg p$
($K_@$)	$\vdash @_{\mathbf{j}}(p \to q) \to (@_{\mathbf{j}} p \to @_{\mathbf{j}} q)$
(*Selfdual*)	$\vdash \neg @_{\mathbf{j}} p \leftrightarrow @_{\mathbf{j}} \neg p$
(*Ref*)	$\vdash @_{\mathbf{j}} \mathbf{j}$
(*Intro*)	$\vdash \mathbf{j} \wedge p \to @_{\mathbf{j}} p$
(*Back*)	$\vdash \Diamond @_{\mathbf{j}} p \to @_{\mathbf{j}} p$
(*Agree*)	$\vdash @_{\mathbf{i}} @_{\mathbf{j}} p \to @_{\mathbf{j}} p$

The logics $\mathbf{H}(@)$ and $\mathbf{H}^+(@)$: inference rules

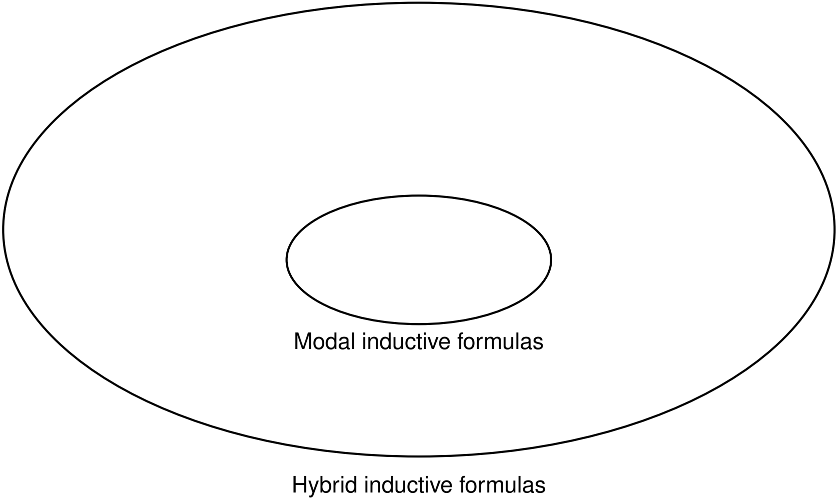
(*Modus ponens*)	If $\vdash \varphi \to \psi$ and $\vdash \varphi$, then $\vdash \psi$.
(*Sorted substitution*)	$\vdash \varphi'$ whenever $\vdash \varphi$, where φ' is obtained from φ by sorted substitution.
(*Nec*)	If $\vdash \varphi$, then $\vdash \Box\varphi$.
(*Nec*$_@$)	If $\vdash \varphi$, then $\vdash @_{\mathbf{j}}\varphi$.
(*Name*$_@$)	If $\vdash @_{\mathbf{j}}\varphi$, then $\vdash \varphi$ for $\mathbf{j}$ not occurring in φ.
(*BG*$_@$)	If $\vdash @_{\mathbf{i}}\Diamond\mathbf{j} \land @_{\mathbf{j}}\varphi \to \psi$, then $\vdash @_{\mathbf{i}}\Diamond\varphi \to \psi$ for $\mathbf{j} \neq \mathbf{i}$ and $\mathbf{j}$ not occurring in φ and ψ.

Syntactic classes



Hybrid inductive formulas

Syntactic classes



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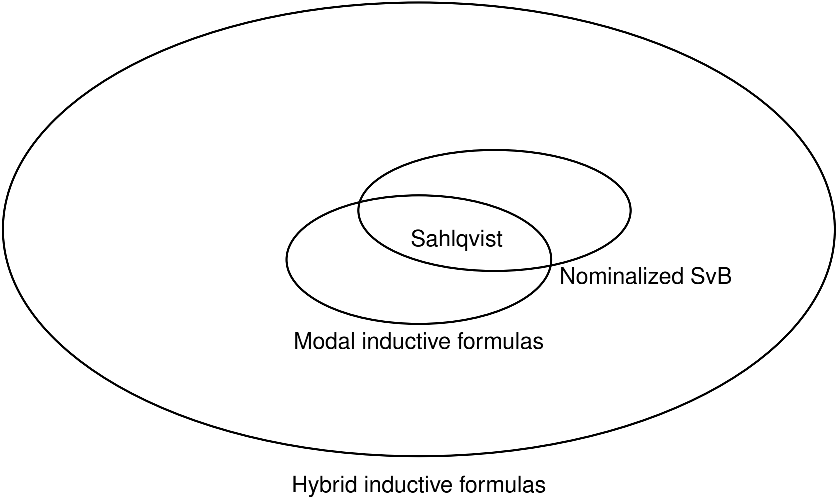

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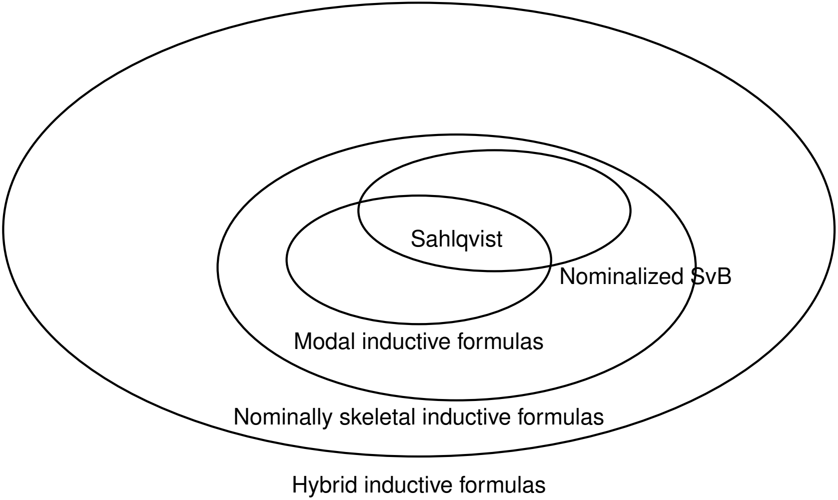

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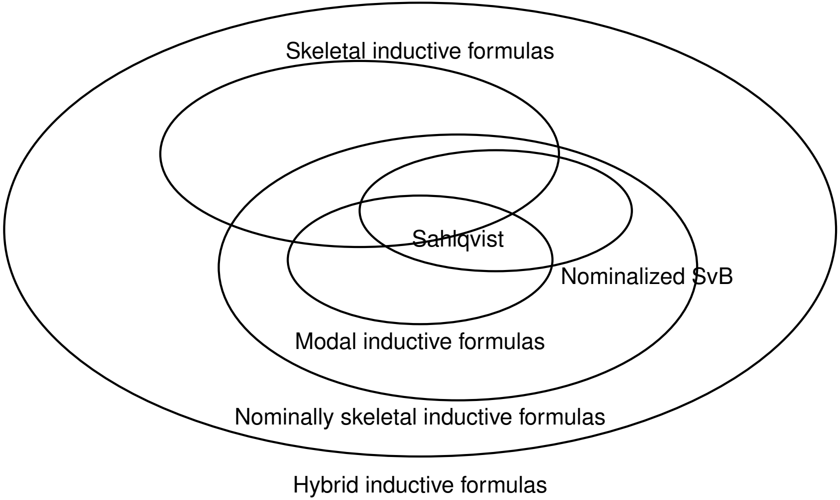

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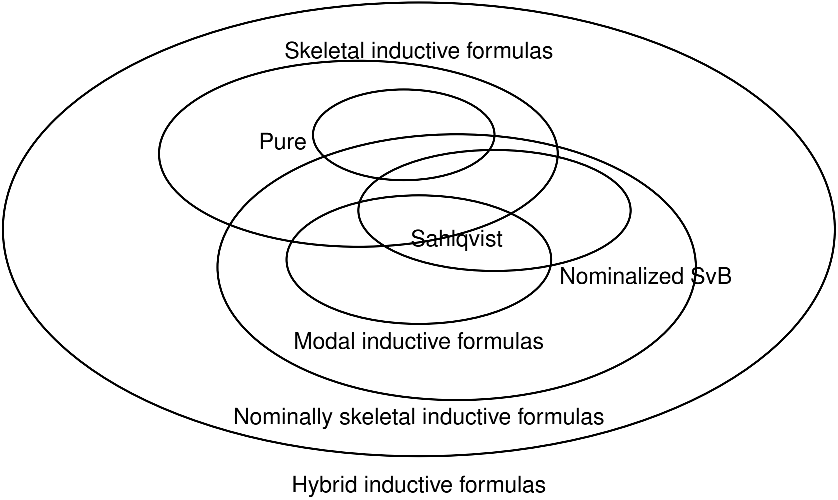

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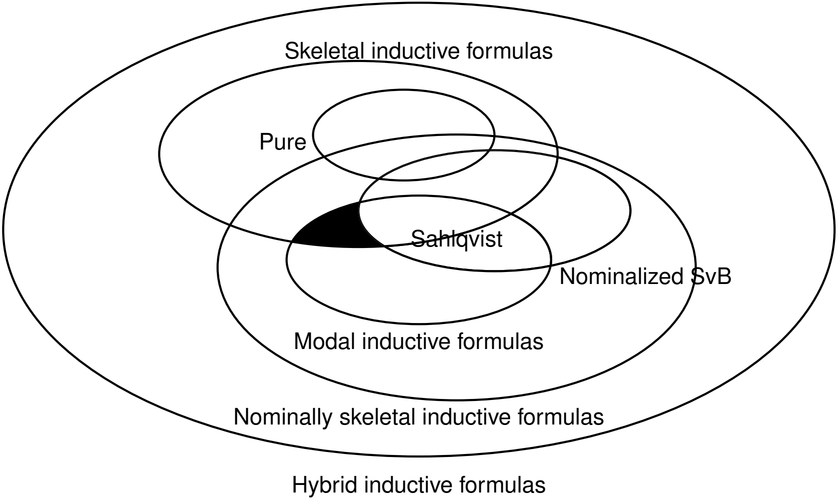
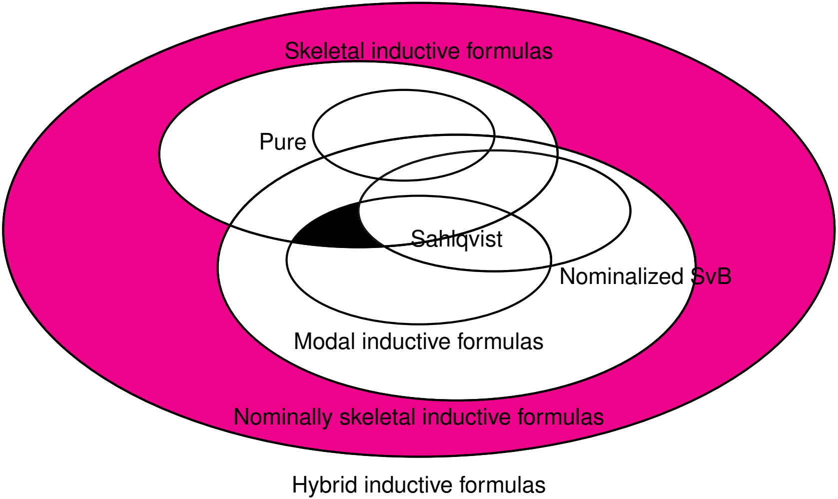


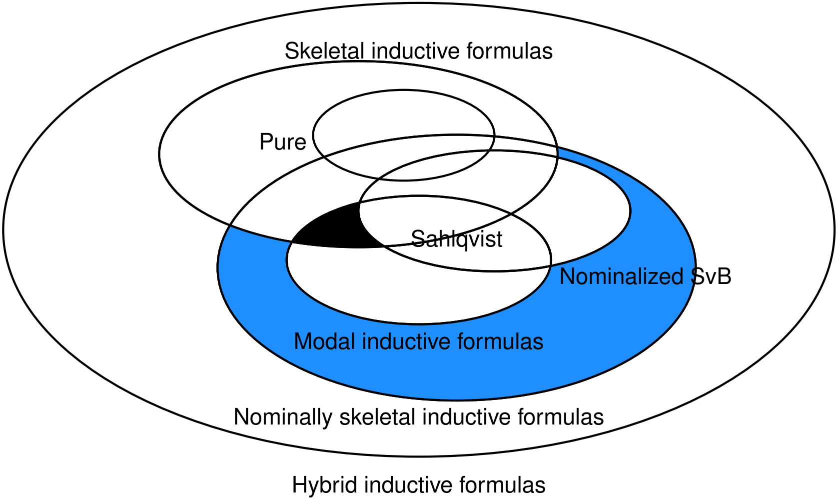
Syntactic classes

$@_{\mathbf{i}} p \wedge \Diamond\Box(p \wedge \mathbf{i}) \rightarrow \Box\Diamond(p \vee \mathbf{i})$



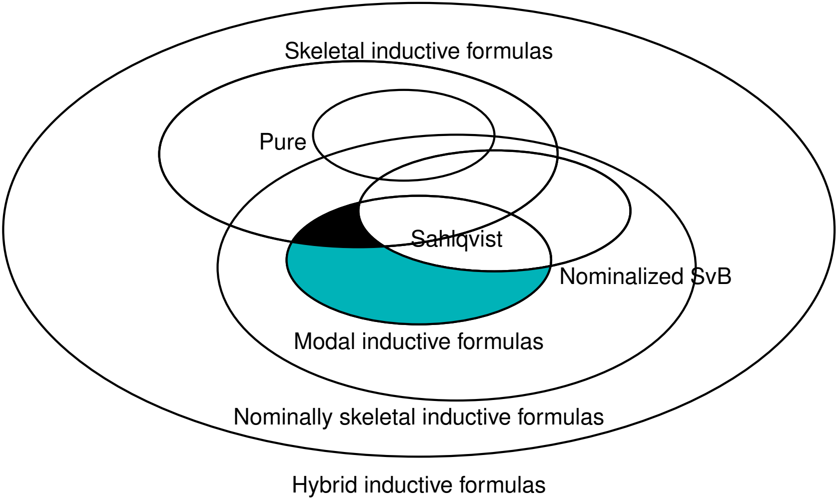
Syntactic classes

$@_{\mathbf{i}}\neg p \wedge \Box(\Box q \to p) \to @_{\mathbf{i}}\Diamond\neg q$



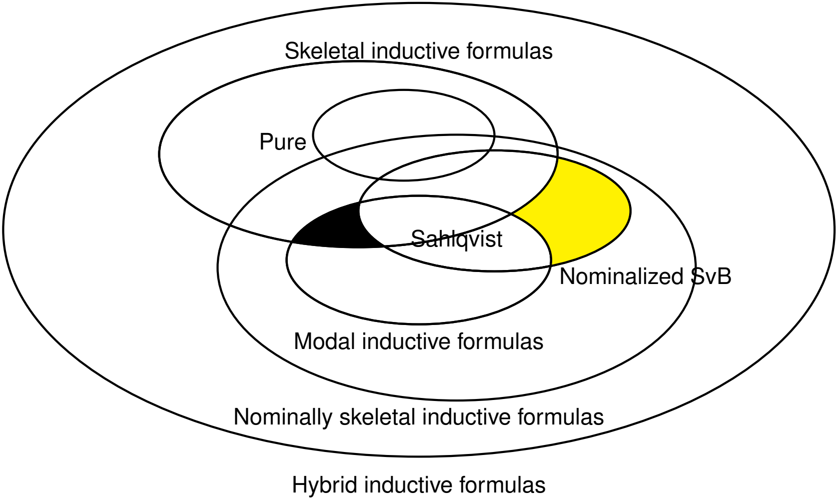
Syntactic classes

$p \wedge \Box(\Diamond p \to \Box q) \to \Diamond\Box\Box q$



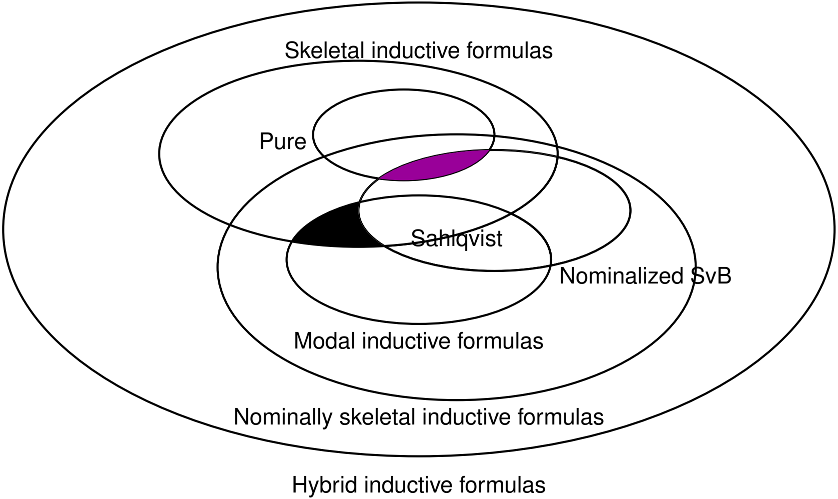
Syntactic classes

$\Box(\neg \mathbf{i} \vee \Diamond \neg p) \vee \Diamond(\mathbf{i} \vee \Box p)$



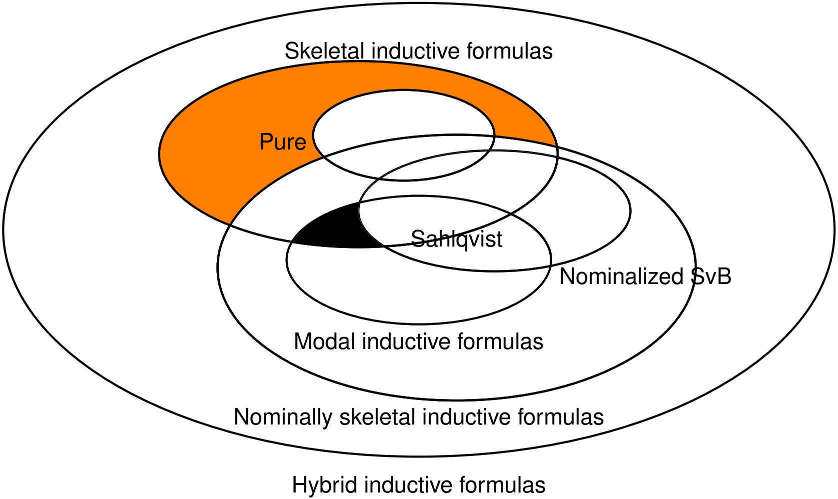
Syntactic classes

$\Diamond \mathbf{i} \rightarrow \mathbf{i}$



Syntactic classes

$\Diamond(p \wedge \Box\mathbf{i}) \rightarrow \Box(p \vee \Diamond\mathbf{i})$



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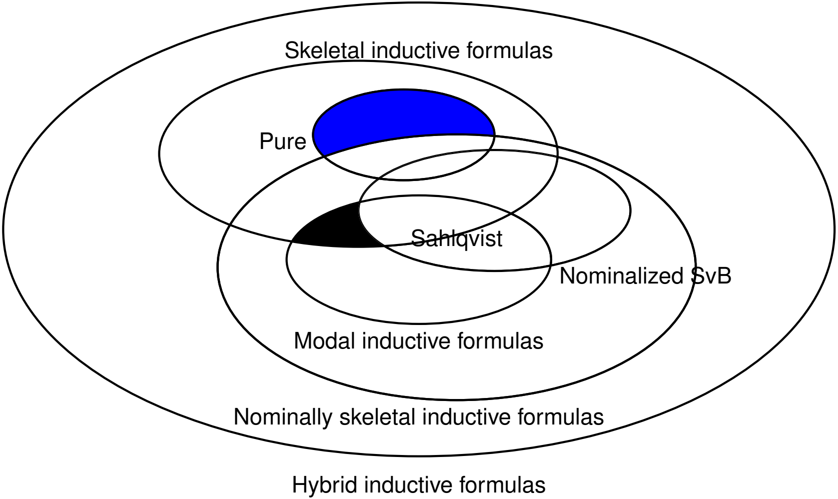
$\Diamond\Box \mathbf{i} \rightarrow \mathbf{i}$



Hybrid-ALBA

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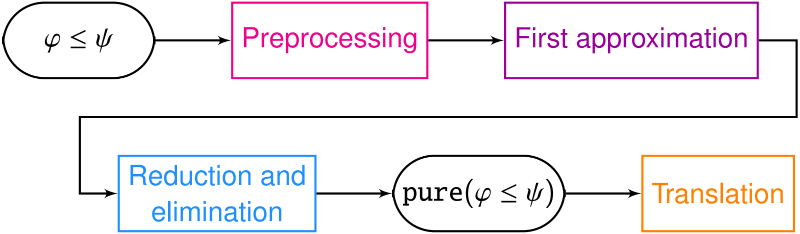

Preprocessing

$$\frac{\alpha \leq \beta \wedge \gamma}{\alpha \leq \beta \ \ \& \ \ \alpha \leq \gamma} \ (\wedge\text{-Adj}) \qquad \frac{\alpha \vee \beta \leq \gamma}{\alpha \leq \gamma \ \ \& \ \ \beta \leq \gamma} \ (\vee\text{-Adj})$$

$$\alpha \wedge (\beta \vee \gamma) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma) \qquad \alpha \vee (\beta \wedge \gamma) \equiv (\alpha \vee \beta) \wedge (\alpha \vee \gamma)$$

$$\neg(\alpha \vee \beta) \equiv \neg\alpha \wedge \neg\beta \qquad \neg(\alpha \wedge \beta) \equiv \neg\alpha \vee \neg\beta$$

$$\Diamond(\alpha \vee \beta) \equiv \Diamond\alpha \vee \Diamond\beta \qquad \Box(\alpha \wedge \beta) \equiv \Box\alpha \wedge \Box\beta$$

$$@_{\mathbf{i}}(\alpha \vee \beta) \equiv @_{\mathbf{i}}\alpha \vee @_{\mathbf{i}}\beta \qquad @_{\mathbf{i}}(\alpha \wedge \beta) \equiv @_{\mathbf{i}}\alpha \wedge @_{\mathbf{i}}\beta$$

First approximation

Let $\{\varphi_i \leq \psi_i \mid i \in I\}$ be the set of inequalities obtained by preprocessing. Then the following rule is applied to each $\varphi_i \leq \psi_i$ only once:

$$\frac{\varphi_i \leq \psi_i}{\mathbf{i}_0 \leq \varphi_i \ \& \ \psi_i \leq \neg \mathbf{j}_0 \Rightarrow \mathbf{i}_0 \leq \neg \mathbf{j}_0} \text{ (First-Approx)}$$

Here $\mathbf{i}_0$ and $\mathbf{j}_0$ are new nominals which do not occur in any inequality received in input.

Reduction and elimination

Ackermann rules

$$\frac{\&_{i=1}^{n} \alpha_i \leq p \ \ \& \ \ \&_{j=1}^{m} \beta_j(p) \leq \gamma_j(p)}{\&_{j=1}^{m} \beta_j(\bigvee_{i=1}^{n} \alpha_i) \leq \gamma_j(\bigvee_{i=1}^{n} \alpha_i)} \text{ (RH-Ack)}$$

$$\frac{\&_{i=1}^{n} p \leq \alpha_i \ \ \& \ \ \&_{j=1}^{m} \gamma_j(p) \leq \beta_j(p)}{\&_{j=1}^{m} \gamma_j(\bigwedge_{i=1}^{n} \alpha_i) \leq \beta_j(\bigwedge_{i=1}^{n} \alpha_i)} \text{ (LH-Ack)}$$

Here

1. the α_i are p-free,
2. the β_j are positive in p, and
3. the γ_i are negative in p.

Reduction and elimination

Adjunction rules

$$\frac{\alpha \leq \beta \wedge \gamma}{\alpha \leq \beta \ \ \& \ \ \alpha \leq \gamma} \text{ ($\wedge$-Adj)} \qquad \frac{\alpha \vee \beta \leq \gamma}{\alpha \leq \gamma \ \ \& \ \ \beta \leq \gamma} \text{ ($\vee$-Adj)}$$

$$\frac{\alpha \leq \Box\beta}{\Diamond^{-1}\alpha \leq \beta} \text{ ($\Box$-Adj)} \qquad \frac{\Diamond\alpha \leq \beta}{\alpha \leq \Box^{-1}\beta} \text{ ($\Diamond$-Adj)}$$

$$\frac{\alpha \leq \neg\beta}{\beta \leq \neg\alpha} \text{ ($\neg$-R-Adj)} \qquad \frac{\neg\alpha \leq \beta}{\neg\beta \leq \alpha} \text{ ($\neg$-L-Adj)}$$

Reduction and elimination

Residuation rules

$$\frac{\alpha \wedge \beta \leq \gamma}{\alpha \leq \beta \rightarrow \gamma}\ (\wedge\text{-Res}) \qquad \frac{\alpha \leq \beta \vee \gamma}{\alpha \wedge \neg\beta \leq \gamma}\ (\vee\text{-Res}) \qquad \frac{\alpha \leq \beta \rightarrow \gamma}{\alpha \wedge \beta \leq \gamma}\ (\rightarrow\text{-Res})$$

$$\frac{\alpha \leq @_{\mathbf{j}}\beta}{\alpha \leq \bot \quad ⅋ \quad \mathbf{j} \leq \beta}\ (@\text{-R-Res}) \qquad \frac{@_{\mathbf{j}}\alpha \leq \beta}{\top \leq \beta \quad ⅋ \quad \alpha \leq \neg\mathbf{j}}\ (@\text{-L-Res})$$

Reduction and elimination

Approximation rules

$$\frac{\Box\alpha \leq \neg\mathbf{i}}{\exists\mathbf{j}(\Box\neg\mathbf{j} \leq \neg\mathbf{i} \ \&\ \alpha \leq \neg\mathbf{j})}\ (\Box\text{-Approx}) \qquad \frac{\mathbf{i} \leq \Diamond\alpha}{\exists\mathbf{j}(\mathbf{i} \leq \Diamond\mathbf{j} \ \&\ \mathbf{j} \leq \alpha)}\ (\Diamond\text{-Approx})$$

$$\frac{\mathbf{i} \leq @_{\mathbf{j}}\alpha}{\mathbf{j} \leq \alpha}\ (@\text{-R-Approx}) \qquad \frac{@_{\mathbf{j}}\alpha \leq \neg\mathbf{i}}{\alpha \leq \neg\mathbf{j}}\ (@\text{-L-Approx})$$

The nominal **j** in ($\Box$-Approx) and ($\Diamond$-Approx) is a new nominal not occuring in the computation thus far.

Main results

Theorem

Every hybrid inductive formula has a local first-order frame correspondent.

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Every inductive formula that is nominally skeletal is preserved under canonical extensions of permeated hybrid algebras.

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Proof

$$
\begin{array}{ccc}
\mathfrak{A} \models \varphi \leq \psi & & \mathfrak{A}^\delta \models \varphi \leq \psi \\
\Updownarrow & & \\
\mathfrak{A}^\delta \models_{\mathfrak{A}} \varphi \leq \psi & & \Updownarrow \\
\Updownarrow & & \\
\mathfrak{A}^\delta \models_{\mathfrak{A}} \mathbf{i}_0 \leq \varphi \;\&\; \psi \leq \mathbf{j}_0 \Rightarrow \mathbf{i}_0 \leq \neg \mathbf{i}_0 & & \mathfrak{A}^\delta \models \mathbf{i}_0 \leq \varphi \;\&\; \psi \leq \mathbf{j}_0 \Rightarrow \mathbf{i}_0 \leq \neg \mathbf{i}_0 \\
\Updownarrow & & \Updownarrow \\
\mathfrak{A}^\delta \models_{\mathfrak{A}} \mathtt{pure}(\varphi \leq \psi) & \Longleftrightarrow & \mathfrak{A}^\delta \models \mathtt{pure}(\varphi \leq \psi)
\end{array}
$$

Main results

Corollary

For any set Σ of inductive formulas that are nominally skeletal, the logic $\mathbf{H}^{+}(@) \oplus \Sigma$ is sound and strongly complete with respect to the class of Kripke frames defined by the first-order correspondents of the axioms in Σ.

Main results

Theorem

Every skeletal formula is preserved under Dedekind MacNeille completions of atomic hybrid algebras in which $\Diamond$ preserves all existing joins.

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Every skeletal formula is preserved under Dedekind MacNeille completions of atomic hybrid algebras in which $\Diamond$ preserves all existing joins.

Proof

$$
\begin{array}{ccc}
\mathfrak{A} \models \varphi \leq \psi & & \mathfrak{A}^{dm} \models \varphi \leq \psi \\
\Updownarrow & & \\
\mathfrak{A}^{dm} \models_{\mathfrak{A}} \varphi \leq \psi & & \Big\Updownarrow \\
\Updownarrow & & \\
\mathfrak{A}^{dm} \models_{\mathfrak{A}} \mathbf{i}_0 \leq \varphi \ \& \ \psi \leq \mathbf{j}_0 \Rightarrow \mathbf{i}_0 \leq \neg \mathbf{i}_0 & & \mathfrak{A}^{dm} \models \mathbf{i}_0 \leq \varphi \ \& \ \psi \leq \mathbf{j}_0 \Rightarrow \mathbf{i}_0 \leq \neg \mathbf{i}_0 \\
\Updownarrow & & \Updownarrow \\
\mathfrak{A}^{dm} \models_{\mathfrak{A}} \mathtt{pure}(\varphi \leq \psi) & \Longleftrightarrow & \mathfrak{A}^{dm} \models \mathtt{pure}(\varphi \leq \psi)
\end{array}
$$

Main results

Corollary

For any set Σ of skeletal formulas, the logic $\mathbf{H}^{+}(@) \oplus \Sigma$ is sound and strongly complete with respect to the class of Kripke frames defined by the first-order correspondents of the axioms in Σ.

Relationship between main results

- Gehrke, Harding and Venema showed that all varieties of monotone bounded lattice expansions which are closed under MacNeille completions are also closed under canonical extensions.

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- Gehrke, Harding and Venema showed that all varieties of monotone bounded lattice expansions which are closed under MacNeille completions are also closed under canonical extensions.
- So are the skeletal inductive hybrid formulas also preserved under canonical extensions?
- No, the irreflexivity axiom $\mathbf{i} \rightarrow \neg\Diamond\mathbf{i}$ is a skeletal inductive formula which is not preserved under canonical extensions [Conradie].

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Thank you!