Lax orthogonal factorisation systems in Topology

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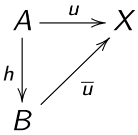
Continuous lattices

A $T0$-space X is said to be injective if any continuous map $u : A \to X$ can be extended along any embedding $h : A \to B$:



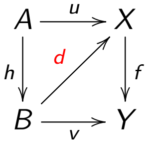
Theorem. A $T0$-space is injective if and only if

- X is a continuous lattice [Scott 1970]
- X has an algebra structure for the filter monad. [Day 1975]

Fibrewise continuous lattices

Question:
Are there similar characterizations for injective continuous maps?

A continuous map $f : X \to Y$ between $T0$-spaces is said to be injective if it is injective as an object of the comma category $\mathbf{Top}_0 \downarrow Y$; that is,



Using Kock-Zöberlein monads

Let **C** be a (pre)ordered enriched category.

Let $\mathbb{T} = (T, \eta, \mu)$ be a Kock-Zöberlein monad on **C**.
The following assertions are equivalent, for a **C**-object X:

- X is injective with respect to T-embeddings,
- X is Kan-injective with respect to T-embeddings,
- X has a $\mathbb{T}$-algebra structure.

[M. Escardó, Properly injective spaces and function spaces. Top. Appl. (1998)]

[M. Escardó, R. Flagg, Semantic domains, injective spaces and monads, Elect. Notes TCS (1999)]

Using Kock-Zöberlein monads

Let **C** be a (pre)ordered enriched category.

Let $\mathbb{T} = (T, \eta, \mu)$ be a Kock-Zöberlein (KZ) monad on **C**.
The following assertions are equivalent, for a **C**-object X:

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A monad $\mathbb{T} = (T, \eta, \mu)$ is KZ if the (equivalent) conditions hold:

(i) $T\eta \leq \eta_T$;

(ii) $T\eta_X \dashv \mu_X$ for every X;

(iii) $\mu_X \dashv \eta_T$ for every X;

(iv) $TX \xrightarrow{a} X$ is a $\mathbb{T}$-algebra iff $TX \underset{a}{\overset{\eta_X}{\rightleftarrows}} X$ ($\top$) with id counit.

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A morphism $h : A \to B$ is a T-embedding if

$$TA \underset{T^*h}{\overset{Th}{\rightleftarrows}} TB \quad (\bot) \qquad \text{with id unit;}$$

that is: $1_{TA} = T^*h \cdot Th$ and $Th \cdot T^*h \leq 1_{TB}$.

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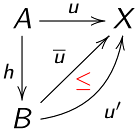
An object X is Kan-injective with respect to $h : A \to B$ if any $u : A \to X$ can be extended along $h : A \to B$ in a universal way:



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Consider in Top_0 the order: $x \leq y$ if $y \in \overline{\{x\}}$;
that is, the dual of the specialisation order.
The filter monad $\mathbb{F} = (F, \eta, \mu)$

- is Kock-Zöberlein,
- F-embeddings are embeddings, and so
- Scott's result follows from Day's result.

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Fibrewising Escardó's results

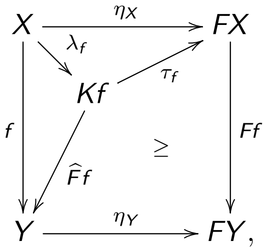
Question: Can we use Escardó's results for continuous maps?

Difficulties:

- Construction of a fibrewise filter monad $\widehat{\mathbb{F}}$
- Do the $\widehat{\mathbb{F}}$-embeddings coincide with embeddings (as for $\mathbb{F}$)?

Fibrewise Injectivity

For each $T0$-space Y, the fibrewise filter monad $\widehat{\mathbb{F}} = (\widehat{F}, \lambda, \hat{\mu})$ on $\mathbf{Top}_0 \downarrow Y$, defined using comma objects:



where:

- $Kf = \{(y, \varphi) \in Y \times FY\,;\, Ff(\varphi) \leq \mathcal{O}(y)\}$
- $\widehat{F}(f)$ and τ_f are projections, and $\lambda_f(x) = (f(x), \mathcal{O}(x))$,

[Cagliary, C, Mantovani, Fibrewise injectivity and Kock-Zöberlein monads, JPAA 2012]

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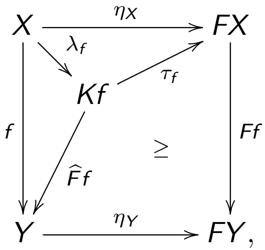

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- $\widehat{F}(f)$ and τ_f are projections, and $\lambda_f(x) = (f(x), \mathcal{O}(x))$,

is a Kock-Zöberlein monad,
such that embeddings coincide with $\widehat{F}$-embeddings.

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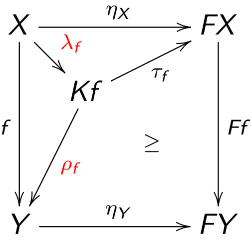
The weak factorisation system

The factorisation of f as



gives a weak factorisation system $(\mathcal{L}, \mathcal{R})$ in $\mathbf{Top_0}$, with
$\mathcal{L} = \{\text{embeddings}\} = \{F\text{-embeddings}\} = \{\widehat{F}\text{-embeddings}\}$, and
$\mathcal{R} = \{\text{injective continuous maps}\} = \{\widehat{\mathbb{F}}\text{-algebras}\}$;

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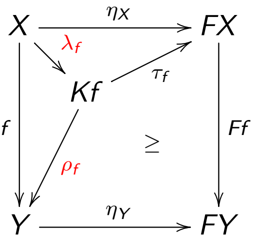

gives a weak factorisation system $(\mathcal{L}, \mathcal{R})$ in $\mathbf{Top_0}$, with $\mathcal{L} = \{\text{embeddings}\}$ and $\mathcal{R} = \{\text{injective continuous maps}\}$; that is,

- every continuous maps can be written as a composition of a morphism in $\mathcal{L}$ followed by one in $\mathcal{R}$,

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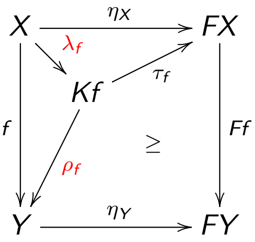

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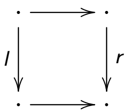
- every continuous maps can be written as a composition of a morphism in $\mathcal{L}$ followed by one in $\mathcal{R}$,
- for any commutative square  , with $l \in \mathcal{L}$ and $r \in \mathcal{R}$

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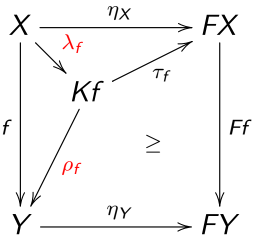

gives a weak factorisation system $(\mathcal{L}, \mathcal{R})$ in $\mathbf{Top_0}$, with $\mathcal{L} = \{\text{embeddings}\}$ and $\mathcal{R} = \{\text{injective continuous maps}\}$; that is,

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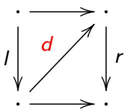
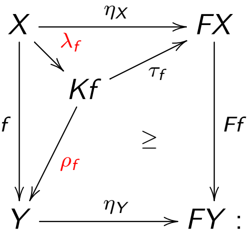
- for any commutative square 

, with $l \in \mathcal{L}$ and $r \in \mathcal{R}$, there exists a diagonal filler d.

[Cagliary, C, Mantovani, Fibrewise injectivity and Kock-Zöberlein monads, JPAA 2012]

Special properties of this factorisation system

The factorisation of f as



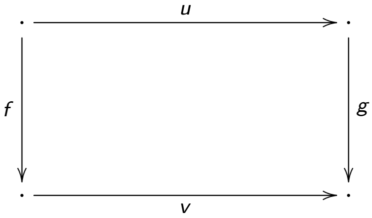
- gives a functorial factorisation system $\mathbf{Top}_0^2 \to \mathbf{Top}_0^3$;
- the endofunctors $L : f \mapsto \lambda_f$ and $R : f \mapsto \rho_f$ can be endowed resp. with a comonad $\mathbb{L}$ and a monad $\mathbb{R}$ structures, satisfying a distributivity law: $(\mathbb{L}, \mathbb{R})$ is an algebraic factorisation system.
- both $\mathbb{L}$ and $\mathbb{R}$ are Kock-Zöberlein.

Note that an orthogonal factorisation system is an algebraic one $(\mathbb{L}, \mathbb{R})$ with $\mathbb{L}$ and $\mathbb{R}$ idempotent.

[C, López-Franco, Lax orthogonal factorisation systems, arXiv 1503.06469]

Lax orthogonal factorisation systems

An algebraic f.s. $(\mathbb{L}, \mathbb{R})$ is lax orthogonal if both $\mathbb{L}$ and $\mathbb{R}$ are KZ. For each commutative diagram as below, if f is an $\mathbb{L}$-coalgebra and g an $\mathbb{R}$-algebra, the diagonal filler d is obtained as:



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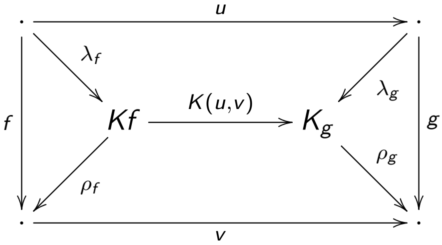

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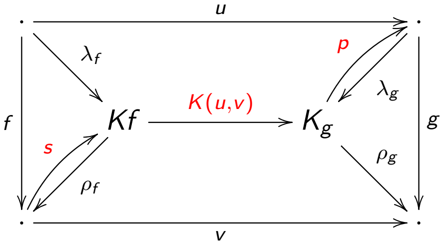

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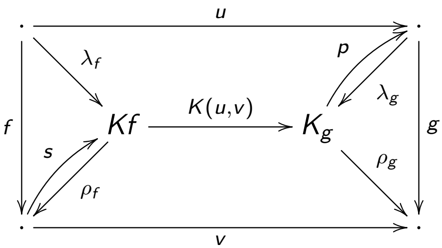
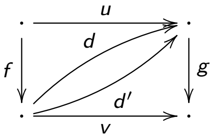


Moreover, this diagonal filler has a universal property:



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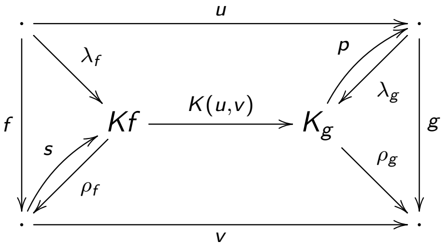

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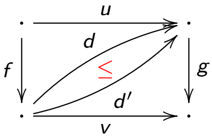

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Examples: Filter monads in $\mathbf{Top}_0$

Escardó-Flagg examples include the following filter monads:

Monad $\mathbb{T}$	T-embeddings	$\mathbb{T}$-algebras= Inj objects wrt T-emb
Filters	Embeddings	Continuous lattices
Proper filters	Dense embeddings	Continuous Scott domains
Prime filters	Flat embeddings	Stably compact spaces
Compl. prime filters	Completely flat emb	Sober spaces

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All these examples can be fibrewised, using the facts:

- $\widehat{\mathbb{F}}$ is KZ, and $\widehat{F}$-embeddings=F-embeddings=embeddings
- the other monads are *well-behaved* submonads of $\mathbb{F}$.

[Cagliary, C, Mantovani, Fibrewise injectivity and Kock-Zöberlein monads, JPAA 2012]

[C, López-Franco, Lax orthogonal factorisation systems in Topology, in preparation]

Examples: Fibrewise filter monads in $\mathbf{Top}_0$

Monad $\widehat{\mathbb{T}}$	$\widehat{T}$-embeddings	$\widehat{\mathbb{T}}$-algebras= Inj morph wrt $\widehat{T}$-embeddings
Fib filters	Embeddings	Fib continuous lattices
Fib proper filters	Dense embeddings	Fib continuous Scott domains
Fib prime filters	Flat embeddings	Fib stably compact spaces
Fib compl prime f.	Completely flat emb	Fib sober spaces

- How can these $\widehat{\mathbb{T}}$-algebras be characterized?

[Cagliary, C, Mantovani, Fibrewise injectivity and Kock-Zöberlein monads, JPAA 2012]

Examples: Fibrewise filter monads in $\mathbf{Top}_0$

If $\mathbb{T}$ is a submonad of $\mathbb{F}$, define the $\mathbb{T}$-way below relation $\ll^{\mathbb{T}}$ as:

$$V \ll^{\mathbb{T}} U \;:\Leftrightarrow\; (\forall \varphi \in TX)\,(V \in \varphi \;\Rightarrow\; \lim \varphi \cap U \neq \emptyset),$$

for all $T0$-spaces X and $U, V \in \mathcal{O}X$.

[Hofmann, A four for the price of one duality principle for distributive topological spaces, Order 2013]

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Theorem. For a $T0$-space X the following are equivalent:

(i) X is a $\mathbb{T}$-algebra,

(ii) X is sober, $\mathbb{T}$-corecompact and $\mathbb{T}$-stable.

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A $T0$-space is $\mathbb{T}$-corecompact if

$$(\forall U \in \mathcal{O}X)\;\; U = \bigvee \{V\,;\, V \in \mathcal{O}X,\; V \ll^{\mathbb{T}} U\}.$$

A $T0$-space is $\mathbb{T}$-stable if

$$\forall J \text{ finite}, (U_j)_j, (V_j)_j \text{ in } \mathcal{O}X\; (\forall j \in J\;\; V_j \ll^{\mathbb{T}} U_j) \;\Rightarrow\; \bigcap_{j\in J} V_j \ll^{\mathbb{T}} \bigcap_{j\in J} U_j.$$

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Examples: Fibrewise filter monads in $\mathbf{Top}_0$

If $\mathbb{T}$ is a submonad of $\mathbb{F}$, the fibrewise $\mathbb{T}$-way below relation:

$$V \ll^{\mathbb{T}}_{W} U \;:\Leftrightarrow\; (\forall (z,\varphi) \in W \times V^{\sharp} \subseteq \widehat{T}X)\ \lim \varphi \cap X_z \cap U \neq \emptyset,$$

for continuous maps $f : X \to Y$ in $\mathbf{Top_0}$, $U, V \in \mathcal{O}X$, $W \in \mathcal{O}Y$.

[Cagliary, C, Mantovani, Fibrewise injectivity and Kock-Zöberlein monads, JPAA 2012]

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Examples: Fibrewise filter monads in $\mathbf{Top}_0$

If $\mathbb{T}$ is a submonad of $\mathbb{F}$, the fibrewise $\mathbb{T}$-way below relation:

$$V \ll_W^{\mathbb{T}} U \;:\Leftrightarrow\; (\forall (z,\varphi) \in W \times V^\sharp \subseteq \widehat{T}X)\ \lim \varphi \cap X_z \cap U \neq \emptyset,$$

for continuous maps $f : X \to Y$ in $\mathbf{Top}_0$, $U, V \in \mathcal{O}X$, $W \in \mathcal{O}Y$.

Theorem.
For a continuous map $f : X \to Y$ the following are equivalent:

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(i) f is a $\widehat{\mathbb{T}}$-algebra,

(ii) f is fibrewise sober, $\widehat{\mathbb{T}}$-corecompact and $\widehat{\mathbb{T}}$-stable.

A continuous map $f : X \to Y$ is:

- fibrewise sober if, for irreducible closed subsets A of X and $y \in Y$,
$$\overline{f(A)} = \overline{\{y\}} \;\Rightarrow\; \exists! x \in X_y \;:\; A = \overline{\{x\}}.$$
- fibrewise $\mathbb{T}$-corecompact if, for all $U \in \mathcal{O}X$,
$$U = \bigvee \{ V \cap X_W \,;\, V \in \mathcal{O}X,\; W \in \mathcal{O}Y,\; V \cap X_W \ll^{\mathbb{T}}_W U \cap X_W \}.$$
- fibrewise $\mathbb{T}$-stable if
$$\forall J \text{ finite}, (U_j)_j, (V_j)_j \text{ in } \mathcal{O}X \; (\forall j \in J \;\; V_j \ll^{\mathbb{T}}_{W_j} U_j) \;\Rightarrow\; \bigcap_{j \in J} V_j \ll^{\mathbb{T}}_W \bigcap_{j \in J} U_j,$$
where $W_j \in \mathcal{O}Y$ and $W = \cap_{j \in J} W_j$.

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Theorem.

For a continuous map $f : X \to Y$ the following are equivalent:

(i) f is a $\widehat{\mathbb{T}}$-algebra,

(ii) f is fibrewise sober, fibrewise $\widehat{\mathbb{T}}$-corecompact and fibrewise $\widehat{\mathbb{T}}$-stable.

Examples of lax orthogonal factorisation systems in $\mathbf{Top}_0$:
(T-embeddings, $\widehat{\mathbb{T}}$-algebras), when $\mathbb{T}$ is

- the filter monad,
- the proper filter monad,
- the prime filter monad,
- the completely prime filter monad.

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Lax orthogonal factorization systems in $(\mathbb{T}, V)$-spaces

First example:

The Cauchy completion of a (generalised) metric space

[Lawvere, Metric spaces, generalized logic, and closed categories, Rend. Sem. M. F. Milano 1973; TAC Rep. 2002]

[C, Hofmann, Relative injectivity as cocompleteness for a class of distributors, TAC 2008]

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Lax orthogonal factorization systems in $(\mathbb{T}, V)$-spaces

General construction of examples:

In the setting of [C, Hofmann, TAC 2008], given a quantale V, a **Set**-monad $\mathbb{T}$ conveniently extended to V-**Rel**, and a class of bimodules (or distributors) Φ,

- the category $(\mathbb{T}, V)$-**Cat** is order enriched,
- the presheaf monad P on $(\mathbb{T}, V)$-**Cat** is KZ, as well as the corresponding fibrewise presheaf monad,
- embeddings=P-embeddings=$\widehat{P}$-embeddings,
- the submonads Φ of P are *well-behaved*,

so that each choice of a class Φ gives rise to a pair $(\mathbb{L}, \mathbb{R})$ of a KZ comonad $\mathbb{L}$ and a KZ monad $\mathbb{R}$, that is, a lax factorisation system.

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