An Abstract Algebraic Logic View on Judgment Aggregation

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Social Choice Theory

Theoretical study of **collective decision making**

- Main question:
  - how to combine individual opinions (preferences, judgments...) into **one** collective opinion (preference, judgment) in a **fair** way?

- Examples:
  - Political elections
  - Judgments in a court of law
  - Opinion poolings
  - ...
- Various branches:
  - Preference aggregation
  - Judgment aggregation
  - Opinion pooling
  - ...

The beginning of modern social choice theory

C: set of candidates

N: set of individuals

L: set of linear orders on C

$f : L^N \to L$ social welfare function (SWF)

Properties of SWFs:

- Pareto
- independence of irrelevant alternatives (IIA)
- dictatorship

Arrow's Impossibility Theorem

Let $|N| < \omega$, $|C| \geq 3$. Then

$$\text{Pareto, IIA} \models \text{dictatorship}.$$

The Ultrafilter Method

- Decisive coalitions
- Pareto and IIA $\Longrightarrow$ decisive coalitions form an (ultra)filter
  - Properness, upward directedness $\surd$
  - intersection property: non-trivial!

If N finite,

- all ultrafilters are principal $\Longrightarrow$ **one** generator: the dictator
- all filters are **finitely** generated $\Longrightarrow$ the oligarchs

Generalisations

Technically:

- Impossibility theorems $\Rightarrow$ **characterization** theorems
- Ultrafilter argument applies to
  - judgment aggregation
  - opinion pooling
- Model-theoretic approach: rational aggregators as **ultraproducts**
- Algebraic approach: rational aggregators as (Boolean algebras, MV-algebras) **homomorphisms**

Philosophically:

The subjunctive interpretation of logical connectives helps to escape the impossibility theorems

A unifying approach: AAL

Abstract Algebraic Logic (AAL)

- theory for uniform algebraization of logics
- Logics: $S = (Fm, \vdash_S)$ (consequence relations: first-class citizens)
- every logic S canonically associated with class of algebras Alg_S
- metalogical properties of S studies via Alg_S

Selfextensional Logics

- Logics S s.t. $\dashv\vdash_S$ is a congruence of $\mathbf{Fm}$.
- Characterised as the logics admitting a **possible world semantics** (subjunctive interpretation of logical connectives)
- Examples: classical, intuitionistic, modal$^{\leq}$, Łukasiewicz$^{\leq}$

Selfextensional logics as natural environment for judgment aggregation!

For any selfextensional logic S

Fm: formulas

$X \subseteq \mathbf{Fm}$: *agenda*

$\bar{X}$: S-closure of X

- $A : X \to \mathbf{B}$: attitude function
- $\vec{A} \in (\mathbf{B}^X)^N$: attitude profile
- $F : (\mathbf{B}^X)^N \to \mathbf{B}^X$: attitude aggregator

- A is *rational* if it can be extended to a homomorphism $\bar{A} : \mathbf{Fm}_{/\equiv} \to \mathbf{B}$ of S-algebras.
- $\vec{A} \in (\mathbf{B}^X)^N$ is *rational* if $(\forall i \in N)(A_i$ is rational).
- F is *rational* if ($\forall$rational $\vec{A} \in dom(F))(F(\vec{A})$ is rational).
- F is *universal* if $dom(F) = (\mathbf{B}^X)^N$.

Decision criteria and systematicity

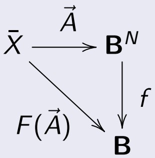
Def. Decision Criterion for F

$f : \mathbf{B}^N \to \mathbf{B}$ s.t. $\forall \vec{A} \in dom(F)$, $\forall \varphi \in X$,

$$F(\vec{A})(\varphi) = f(\vec{A}(\varphi)). \tag{1}$$

Strong Systematicity

F is *strongly systematic* if $\exists f$ (decision for F) s.t. $\forall \vec{A} \in dom(F)$,



Main Theorem

Let F be a rational, universal and strongly systematic attitude aggregator. Then the decision criterion of F is a homomorphism of $\mathcal{S}$-algebras.

Let $f : \mathbf{B}^N \to \mathbf{B}$ be a homomorphism of $\mathcal{S}$-algebras. Then the function $F : (\mathbf{B}^X)^N \to \mathbf{B}^X$ defined by $F(\vec{A})(\varphi) = f(\vec{A}(\varphi))$ is a rational, universal and strongly systematic attitude aggregator.