A description of the localic group of the unit circle

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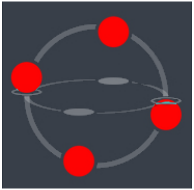

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Universidad del País Vasco
Euskal Herriko Unibertsitatea

[1]Joint work with Javier Gutiérrez García and Jorge Picado

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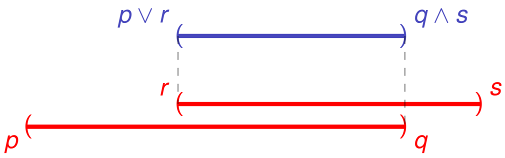

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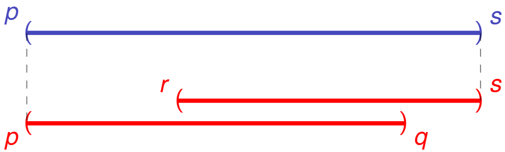

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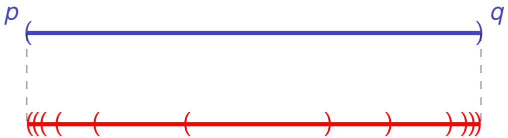

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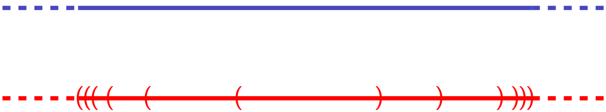

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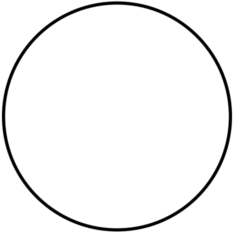
B. Banaschewski,
The Real Numbers in Pointfree Topology,
Textos Mat. Sér. B **12** Universidade de Coimbra (1997).

The quotient space $\mathbb{R}/\mathbb{Z}$



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$\mathfrak{L}(\mathbb{T})$ is isomorphic to the Alexandroff compactification of $\mathfrak{L}(\mathbb{R})$.

$\mathfrak{L}(\mathbb{T})$ as a localic quotient $\mathfrak{L}(\mathbb{R})$

Let $e\colon \mathfrak{L}(\mathbb{T}) \to \mathfrak{L}(\mathbb{R})$ be given by

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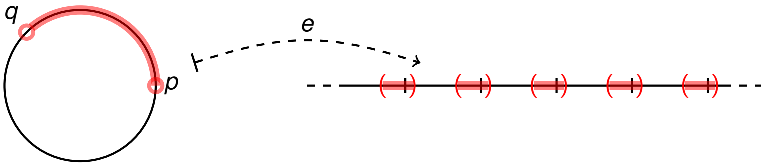

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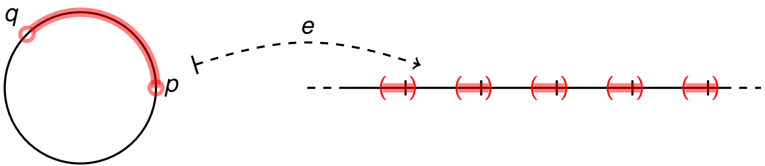

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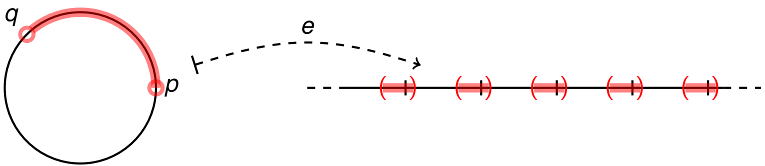

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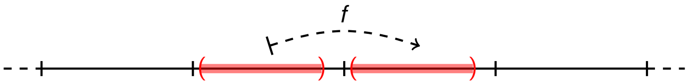

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where $\overline{h}(x) = h(x)$

The localic group of the frame of the real numbers: $(\mathfrak{L}(\mathbb{R}), \mu, \gamma, \varepsilon)$

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Localic groups

$$L \xrightarrow{\mu} L \oplus L$$

Multiplication

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$$L \xrightarrow{\mu} L \oplus L \qquad\qquad L \xrightarrow{\gamma} L$$

Multiplication   Inverse

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$$\begin{array}{ccc} L & \xrightarrow{\mu} & L \oplus L \\ {\scriptstyle \mu}\downarrow & & \downarrow{\scriptstyle \mu \oplus 1_L} \\ L \oplus L & \xrightarrow{1_L \oplus \mu} & L \oplus L \oplus L \end{array}$$

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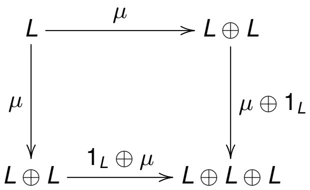

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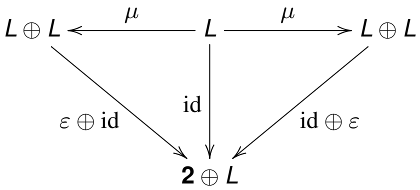

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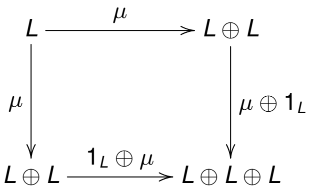

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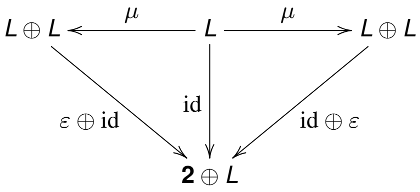

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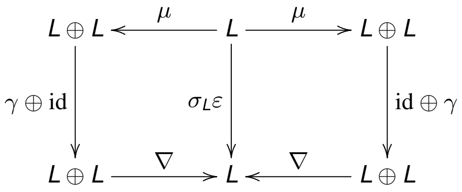

Inverse elements

Can we lift the localic group structure to a quotient?

- Inverse morphism:

$$E \xrightarrow{\quad e \quad} L \underset{g}{\overset{f}{\rightrightarrows}} M \quad \text{an equalizer.}$$

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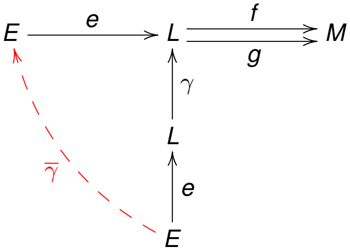

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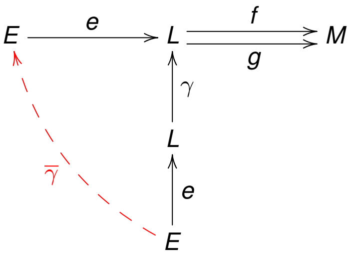

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- Identity morphism:

$$E \xrightarrow{e} L \xrightarrow{\varepsilon} \mathbf{2}$$

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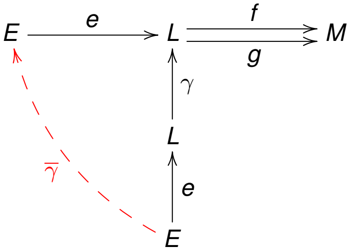

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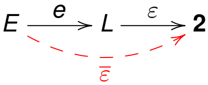

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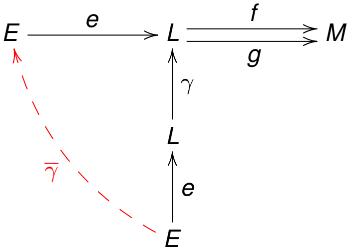

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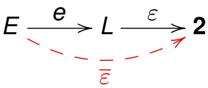

- Multiplication morphism??

$$E \xrightarrow{\;\overline{\mu}\,?\;} E \oplus E$$

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$$E \xrightarrow{\ e\ } L \xrightarrow{\ \gamma\ } L \underset{g}{\overset{f}{\rightrightarrows}} M$$

$$E \xrightarrow{\quad e \quad} L \xrightarrow{\quad \mu \quad} L \oplus L \underset{g \oplus 1_L}{\overset{f \oplus 1_L}{\rightrightarrows}} M \oplus L$$

$$L \oplus L \underset{1_L \oplus g}{\overset{1_L \oplus f}{\rightrightarrows}} L \oplus M$$

Induced localic group structures

- Let $(L, \mu, \gamma, \varepsilon)$ be a localic group
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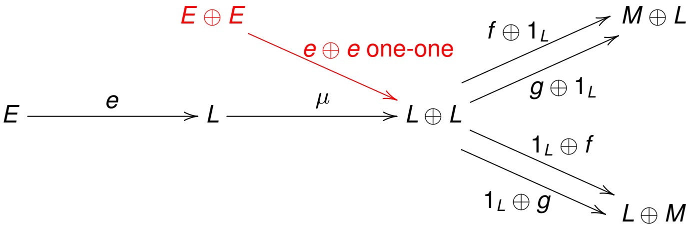

Induced localic group structures

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Theorem

$$(\mathfrak{L}(\mathbb{T}), \overline{\mu}, \overline{\gamma}, \overline{\varepsilon}) \quad \textit{is an induced abelian localic group.}$$

Mila esker!

Grazie mille!

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