GENERALIZING BELNAP'S CUT-ELIMINATION

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TACL - 26 June

OUTLINE

❶ Main Quest

❷ 'Good' Proof Systems: Desiderata

❸ Working Examples: DEL & PDL

❹ Diagnosis & cure

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❻ The multi-type approach

❼ A glimpse at rules for DEL & PDL

❽ Cut rules in Gentzen's Calculi

❾ Canonical cut elimination

❿ Conclusions & Future work

MAIN QUEST

PROOF-THEORY FOR DYNAMIC LOGICS.

Often, the hurdles are due to some of their **defining features** (e.g. lack of closure under uniform substitution).

Typically, these logics come in ***large families***:

- ‘uniform’ proof-theoretic approaches are in high demand.

Ongoing project with S. Frittella, A. Kurz, A. Palmigiano:

- case studies: **DEL**, **PDL**.

‘GOOD’ PROOF SYSTEMS: DESIDERATA

- An **independent** account of dynamic logics:
  - Proof-theoretic semantic approach
- Intuitive, **user-friendly** rules.
- **Good performances**:
  - soundness & completeness,
  - cut-elimination & sub-formula property,
  - decidability.
- A **modular** account of dynamic logics:
  - charting the space of DLs by adding/subtracting rules,
  - transfer of results with minimal changes.

WE1: DYNAMIC EPISTEMIC LOGIC

Interaction axioms: classical case

$$\langle\alpha\rangle p \leftrightarrow \mathit{Pre}(\alpha) \wedge p$$
$$\langle\alpha\rangle(A \vee B) \leftrightarrow \langle\alpha\rangle A \vee \langle\alpha\rangle B$$
$$\langle\alpha\rangle\neg A \leftrightarrow \mathit{Pre}(\alpha) \wedge \neg\langle\alpha\rangle A$$

$$\langle\alpha\rangle\langle\mathsf{a}\rangle A \leftrightarrow \mathit{Pre}(\alpha) \wedge \bigvee\{\langle\mathsf{a}\rangle\langle\beta\rangle A \mid \alpha\mathsf{a}\beta\}$$

Intuitionistic case: more axioms, e.g.

$$\langle\mathsf{a}\rangle(A \to B) \to ([\mathsf{a}]A \to \langle\mathsf{a}\rangle B)$$
$$[\alpha]\langle\mathsf{a}\rangle A \leftrightarrow \mathit{Pre}(\alpha) \to \bigvee\{\langle\mathsf{a}\rangle\langle\beta\rangle A \mid \alpha\mathsf{a}\beta\}$$

WE2: PROPOSITIONAL DYNAMIC LOGIC

Box axioms

$[\alpha](A \rightarrow B) \rightarrow ([\alpha]A \rightarrow [\alpha]B)$

$[\alpha \cup \beta]A \leftrightarrow [\alpha]A \wedge [\beta]A$

$[\alpha ; \beta]A \leftrightarrow [\alpha][\beta]A$

$[?A]B \leftrightarrow (A \rightarrow B)$

$[\alpha](A \wedge B) \leftrightarrow [\alpha]A \wedge [\alpha]B$

$[\alpha^*]A \leftrightarrow A \wedge [\alpha][\alpha^*]A$

$A \wedge [\alpha^*](A \rightarrow [\alpha]A) \rightarrow [\alpha^*]A$

DIAGNOSIS & CURE

$$\textit{swap-out}_L \; \frac{\Big(\mathit{Pre}(\alpha)\,;\{\mathtt{a}\}\{\beta\}\,X \vdash Y \mid \alpha\mathtt{a}\beta\Big)}{\mathit{Pre}(\alpha)\,;\{\alpha\}\{\mathtt{a}\}X \vdash \boldsymbol{;}\Big(Y \mid \alpha\mathtt{a}\beta\Big)}$$

× Diagnosis:

- lack of expressivity
- lack of modularity

✓ Cure:

- add structural connectives
- add types

DISPLAY CALCULI

- Natural generalization of sequent calculi;
- sequents $X \vdash Y$, where X and Y are STRUCTURES:
  - built by **structural connectives** (generalising the role of the comma)
  - **binary trees** (not sequences)
- DISPLAY PROPERTY: **adjunction** at the structural level
- Canonical proof of cut elimination

DC: STRUCTURAL AND LOGICAL CONNECTIVES

Structural connectives are interpreted ***contextually***
(like Gentzen's comma) :

I		$;$		$>$	
$\top$	$\bot$	$\wedge$	$\vee$	$>\!\!\!-$	$\rightarrow$

$\{\texttt{a}\}$		$\overbrace{\underbrace{\texttt{a}}}$		$\{\alpha\}$		$\overbrace{\underbrace{\alpha}}$	
$\langle\texttt{a}\rangle$	$[\texttt{a}]$	$\underset{\vee}{\overset{\wedge}{\texttt{a}}}$	$\overset{\sqcap}{\underset{\sqcup}{\texttt{a}}}$	$\langle\alpha\rangle$	$[\alpha]$	$\underset{\vee}{\overset{\wedge}{\alpha}}$	$\overset{\sqcap}{\underset{\sqcup}{\alpha}}$

DC: THREE GROUPS OF RULES, 1/3

(1) Display Postulates

$$\begin{array}{c} X ; Y \vdash Z \\ \hline\hline Y \vdash X > Z \end{array} \qquad \begin{array}{c} Z \vdash Y ; X \\ \hline\hline Y > Z \vdash X \end{array}$$

Theorem (Display Property)

Each substructure in a display-sequent is ‘displayable’ in precedent or, exclusively, succedent position.

$$\begin{array}{c} Y \vdash X > Z \\ \hline X ; Y \vdash Z \\ \hline Y ; X \vdash Z \\ \hline X \vdash Y > Z \end{array}$$

DC: THREE GROUPS OF RULES, 2/3

(2) Operational Rules

$$\frac{X \vdash A \qquad B \vdash Y}{A \rightarrow B \vdash X > Y} \qquad \frac{Z \vdash A > B}{Z \vdash A \rightarrow B}$$

$$\frac{\{\alpha\} A \vdash X}{\langle\alpha\rangle A \vdash X} \qquad \frac{X \vdash A}{\{\alpha\} X \vdash \langle\alpha\rangle A}$$

$$\frac{A \vdash X}{[\alpha] A \vdash \{\alpha\} X} \qquad \frac{X \vdash \{\alpha\} A}{X \vdash [\alpha] A}$$

DC: THREE GROUPS OF RULES, 3/3

(3) Structural Rules

$$Gri_L \; \begin{array}{c} X > (Y;Z) \vdash W \\ \hline\hline (X > Y);Z \vdash W \end{array} \qquad \begin{array}{c} W \vdash X > (Y;Z) \\ \hline\hline W \vdash (X > Y);Z \end{array} \; Gri_R$$

$$FS^1_L \; \frac{\{a\}X > \{a\}Y \vdash Z}{\{a\}(X > Y) \vdash Z} \qquad \frac{Z \vdash \overbrace{\underbrace{\mathtt{a}}}\, Y > \overbrace{\underbrace{\mathtt{a}}}\, X}{Z \vdash \overbrace{\underbrace{\mathtt{a}}}\,(Y > X)} \; FS^2_R$$

BUT ...

Rules such as the following are problematic:

$$\mathit{swap\text{-}out}_L\ \frac{\Big(\mathit{Pre}(\alpha)\,;\,\{\mathsf{a}\}\{\beta\}\,X \vdash Y \mid \alpha\mathsf{a}\beta\Big)}{\mathit{Pre}(\alpha)\,;\,\{\alpha\}\{\mathsf{a}\}X \vdash \boldsymbol{;}\Big(Y \mid \alpha\mathsf{a}\beta\Big)}$$

$$\frac{\Big(Y \vdash \mathit{Pre}(\alpha) > \{\mathsf{a}\}\{\beta\}\,X \mid \alpha\mathsf{a}\beta\Big)}{\boldsymbol{;}\Big(Y \mid \alpha\mathsf{a}\beta\Big) \vdash \mathit{Pre}(\alpha) > \{\alpha\}\{\mathsf{a}\}X}\ \mathit{swap\text{-}out}_R$$

THE MULTI-TYPE APPROACH: DEL

- Ag Act Fnc Fm;
  - no ancillary symbols; all types are **first-class citizens**;
- Additional expressivity:
  - operational connectives **merging different types** (à la Abramsky, Vickers):

$$
\begin{array}{lcll}
\vartriangle_1, \blacktriangle_1 & : & \mathsf{Act} \times \mathsf{Fm} \to \mathsf{Fm} & \langle \alpha \rangle A \rightsquigarrow \alpha \vartriangle_1 A \\
\vartriangle_2, \blacktriangle_2 & : & \mathsf{Ag} \times \mathsf{Fm} \to \mathsf{Fm} & \langle \mathsf{a} \rangle A \rightsquigarrow \mathsf{a} \vartriangle_2 A \\
\vartriangle_3, \blacktriangle_3 & : & \mathsf{Ag} \times \mathsf{Fnc} \to \mathsf{Act} & \mathsf{a} \vartriangle_3 \alpha
\end{array}
$$

- Modularity: by adding or subtracting types (games, strategies, coalitions) the whole space of dynamic logics can be charted.

  For $1 \leq i \leq 3$,

$\vartriangle_i$		$\blacktriangle_i$		$\vartriangleright_i$		$\blacktriangleright_i$	
$\vartriangle_i$		$\blacktriangle_i$			$-\!\!\vartriangleright_i$		$-\!\!\blacktriangleright_i$

THE MULTI-TYPE APPROACH: PDL

- Act TAct Fm:

$\langle\alpha\rangle A$	becomes	$\alpha \vartriangle_1 A$	$\underset{\vee}{\widehat{\alpha}}\, A$	becomes	$\alpha \blacktriangle_1 A$
$[\alpha]A$	becomes	$\alpha \mathrel{-\!\!\rhd}_1 A$	$\underline{\overline{\alpha}}\, A$	becomes	$\alpha \mathrel{-\!\!\blacktriangleright}_1 A$
$\langle\alpha^+\rangle A$	becomes	$\alpha^+ \vartriangle_0 A$	$\underset{\vee}{\widehat{\alpha^+}}\, A$	becomes	$\alpha^+ \blacktriangle_0 A$
$[\alpha^+]A$	becomes	$\alpha^+ \mathrel{-\!\!\rhd}_0 A$	$\underline{\overline{\alpha^+}}\, A$	becomes	$\alpha^+ \mathrel{-\!\!\blacktriangleright}_0 A$

A GLIMPSE AT RULES FOR DEL

Single-type, first version: rules with side conditions & labels;

$$\textit{swap-out}_L \; \frac{\left(Pre(\alpha)\,;\{\mathtt{a}\}\{\beta\}\,X \vdash Y \mid \alpha \mathtt{a} \beta\right)}{Pre(\alpha)\,;\{\alpha\}\{\mathtt{a}\}X \vdash \mathbf{;}\left(Y \mid \alpha \mathtt{a} \beta\right)}$$

Single-type, emended: purely structural, but labels still there;

$$\textit{swap-out}'_L \; \frac{\left(\Phi_\alpha\,;\{\mathtt{a}\}\{\beta\}\,X \vdash Y \mid \alpha \mathtt{a} \beta\right)}{\Phi_\alpha\,;\{\alpha\}\{\mathtt{a}\}X \vdash \mathbf{;}\left(Y \mid \alpha \mathtt{a} \beta\right)}$$

Multi-type: no side conditions and no labels.

+ (4) Interaction Rules

$$\text{swap-out}_L \; \frac{(\mathtt{a} \blacktriangle F) \blacktriangle (\mathtt{a} \blacktriangle X) \vdash Y}{\mathtt{a} \blacktriangle (F \blacktriangle X) \vdash Y}$$

TUDelft

$$\dfrac{\text{s-out}\ \dfrac{\dfrac{\dfrac{\mathsf{a}\vdash\mathsf{a}\qquad\dfrac{\dfrac{\mathsf{a}\vdash\mathsf{a}\qquad\alpha\vdash\alpha}{\mathsf{a}\blacktriangle\alpha\vdash\mathsf{a}\blacktriangle\alpha}\qquad A\vdash A}{(\mathsf{a}\blacktriangle\alpha)\bigtriangleup A\vdash(\mathsf{a}\blacktriangle\alpha)\vartriangle A}}{\mathsf{a}\bigtriangleup((\mathsf{a}\blacktriangle\alpha)\bigtriangleup A)\vdash\mathsf{a}\vartriangle((\mathsf{a}\blacktriangle\alpha)\vartriangle A)}}{\dfrac{(\mathsf{a}\blacktriangle\alpha)\bigtriangleup A\vdash\mathsf{a}\blacktriangleright(\mathsf{a}\vartriangle((\mathsf{a}\blacktriangle\alpha)\vartriangle A))}{A\vdash(\mathsf{a}\blacktriangle\alpha)\blacktriangleright(\mathsf{a}\blacktriangleright(\mathsf{a}\vartriangle((\mathsf{a}\blacktriangle\alpha)\vartriangle A)))}}}{A\vdash\mathsf{a}\blacktriangleright(\alpha\blacktriangleright(\mathsf{a}\vartriangle((\mathsf{a}\blacktriangle\alpha)\vartriangle A)))}}{\vdots}$$

$$\dfrac{\dfrac{\alpha\bigtriangleup(\alpha\blacktriangle(\alpha\mathrel{-\!\!\rhd}(\mathsf{a}\vartriangle A));\mathrm{I})\vdash\mathsf{a}\vartriangle((\mathsf{a}\blacktriangle\alpha)\vartriangle A)}{(\alpha\mathrel{-\!\!\rhd}(\mathsf{a}\vartriangle A));(\alpha\bigtriangleup\mathrm{I})\vdash\mathsf{a}\vartriangle((\mathsf{a}\blacktriangle\alpha)\vartriangle A)}\ \text{conj}}{\vdots}$$

$$\dfrac{\alpha\mathrel{-\!\!\rhd}(\mathsf{a}\vartriangle A)\vdash\alpha\vartriangle\top\rightarrow\mathsf{a}\vartriangle((\mathsf{a}\blacktriangle\alpha)\vartriangle A)}{[\alpha]\langle\mathsf{a}\rangle A\vdash \mathit{Pre}(\alpha)\rightarrow\bigvee\Big(\langle\mathsf{a}\rangle\langle\beta_i\rangle A\Big)}$$

A glimpse at rules for PDL

Fixed Point Structural Rules

$$FP \vartriangle \frac{\Pi \vartriangle_1 X \vdash Y \qquad (\Pi \,\boldsymbol{;}_3\, \Pi^{\oplus}) \vartriangle_1 X \vdash Y}{\Pi^{\oplus} \vartriangle_0 X \vdash Y}$$

Omega-Iteration Structural Rule

$$\omega \vartriangle \frac{\left(\; \Pi^{(n)} \vartriangle_1 X \vdash Y \;\middle|\; n \geq 1 \; \right)}{\Pi^{\oplus} \vartriangle_0 X \vdash Y}$$

$$
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}{
\color{blue}{\alpha \blacktriangle \alpha \mathrel{-\!\!\triangleright} A \vdash A}
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\dfrac{\alpha \vdash \alpha}{\alpha^{\oplus} \vdash \alpha^{+}}
}{\alpha^{+} \vdash \alpha^{+}}
\qquad A \vdash A
}{\alpha^{+} \mathrel{-\!\!\triangleright} A \vdash \alpha^{+} \triangleright A}
}{\alpha \mathrel{-\!\!\triangleright} (\alpha^{+} \mathrel{-\!\!\triangleright} A) \vdash \alpha \triangleright (\alpha^{+} \triangleright A)}
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}{\alpha^{+} \blacktriangle (\alpha \blacktriangle \alpha \mathrel{-\!\!\triangleright} (\alpha^{+} \mathrel{-\!\!\triangleright} A)) \vdash A}
}{
\color{blue}{(\alpha \,;\, \alpha^{+}) \blacktriangle \alpha \mathrel{-\!\!\triangleright} (\alpha^{+} \mathrel{-\!\!\triangleright} A) \vdash A}
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\color{blue}{\alpha^{+} \blacktriangle (\alpha \mathrel{-\!\!\triangleright} A \,,\, \alpha \mathrel{-\!\!\triangleright} (\alpha^{+} \mathrel{-\!\!\triangleright} A)) \vdash A}
}{\alpha^{+} \blacktriangle (\alpha \mathrel{-\!\!\triangleright} A \,,\, \alpha \mathrel{-\!\!\triangleright} (\alpha^{+} \mathrel{-\!\!\triangleright} A)) \vdash A}
}{\alpha \mathrel{-\!\!\triangleright} A \,,\, \alpha \mathrel{-\!\!\triangleright} (\alpha^{+} \mathrel{-\!\!\triangleright} A) \vdash \alpha^{+} \triangleright A}
}{\alpha \mathrel{-\!\!\triangleright} A \,,\, \alpha \mathrel{-\!\!\triangleright} (\alpha^{+} \mathrel{-\!\!\triangleright} A) \vdash \alpha^{+} \mathrel{-\!\!\triangleright} A}
}{\alpha \mathrel{-\!\!\triangleright} A \wedge \alpha \mathrel{-\!\!\triangleright} (\alpha^{+} \mathrel{-\!\!\triangleright} A) \vdash \alpha^{+} \mathrel{-\!\!\triangleright} A}
}{[\alpha]A \wedge [\alpha][\alpha^{+}]A \vdash [\alpha^{+}]A}
}\; FP \blacktriangle
$$

RESULTS

- Soundness* (DEL, PDL)

  DEL: final coalgebra, PDL: standard relational semantics
- Completeness (DEL, PDL)
- Canonical cut-elimination (DEL, PDL)
- Conservativity (DEL)

∗ Caveat: the virtual adjoints are not interpretable by definition.

CUT RULES IN GENTZEN'S CALCULI

$$\frac{\Gamma \vdash C, \Delta \qquad \Gamma', C \vdash \Delta'}{\Gamma', \Gamma \vdash \Delta', \Delta} \qquad \frac{\Gamma \vdash C, \Delta \qquad \Gamma, C \vdash \Delta}{\Gamma \vdash \Delta} \qquad \frac{\Gamma \vdash C \qquad \Gamma, C \vdash \Delta}{\Gamma \vdash \Delta}$$

$$\frac{\Gamma \vdash C \qquad \Gamma', C \vdash \Delta}{\Gamma', \Gamma \vdash \Delta} \qquad \frac{\Gamma \vdash C, \Delta \qquad C \vdash \Delta'}{\Gamma \vdash \Delta', \Delta} \qquad \frac{\Gamma \vdash C \qquad C \vdash \Delta}{\Gamma \vdash \Delta}$$

Theorem (Cut-elimination)

If $\Gamma \vdash \Delta$ is derivable, then it is derivable without Cut.

- ✓ A Cut is an intermediate step in a deduction.
  'Eliminating the cut' generates a ***new and lemma-free proof***, which employs ***syntactic material coming from the end-sequent***.
- × Typically, syntactic proofs of Cut-elimination are ***non-modular***, i.e. if a new rule is added, it must be proved from scratch.

CANONICAL CUT ELIMINATION, 1/4

Definition

A sequent $x \vdash y$ is ***type-uniform*** if x and y are of the same type.

A (cut) rule is ***strongly type-uniform*** if its premises and conclusion are of the same type.

Theorem (Canonical cut elimination)

If a calculus satisfies the properties below, then it enjoys cut elimination.

CANONICAL CUT ELIMINATION, 2/4

1. structures can disappear, formulas are **forever**;
2. **tree-traceable** formula-occurrences, via suitably defined congruence:
    - same shape, same position, same type, non-proliferation;
3. **principal** = **displayed** (Exception: principal fma's in axioms)
    - Generaliz.: axioms are **closed** under display rules (when applicable);
4. rules are closed under **uniform substitution** of congruent parameters within each type;
5. **reduction strategy** exists when cut formulas are both principal.

SPECIFIC TO MULTI-TYPE SETTING:

6. **type-uniformity** of derivable sequents;
7. **strongly uniform cuts** in each/some type(s).

CANONICAL CUT ELIMINATION, 3/4

Two main cases + subcases.

(a) **Both cut formulas are principal.** by 5. (cut is either eliminated or "broken down" into cuts of lower rank).

(b) **At least one cut formula is parametric.**

- Subcase (b1): a_u principal in axiom. Then,

$$\dfrac{\begin{array}{c}\vdots\,\pi_1\\ x\vdash a\end{array}\qquad\begin{array}{c}(x'\vdash y')[a_u^{pre},a_{suc}]\\ \vdots\,\pi_2\\ a\vdash y\end{array}}{x\vdash y}\qquad\rightsquigarrow\qquad\begin{array}{c}\dfrac{\begin{array}{c}\vdots\,\pi_1\\ x\vdash a\end{array}\qquad a_u\vdash y''[a_{suc}]}{x\vdash y''[a_{suc}]}\\ \vdots\,\pi''\\ (x'\vdash y')[x^{pre},a_{suc}]\\ \vdots\,\pi_2[x/a_u]\\ x\vdash y\end{array}$$

CANONICAL CUT ELIMINATION, 4/4

- Subcase (b2): a_u principal in other rule. Then, a_u is in display, and hence:

$$
\dfrac{\begin{matrix} \vdots\, \pi_1 \\ x \vdash a \end{matrix} \qquad \begin{matrix} \vdots\, \pi_2' \\ a_u \vdash y' \\ \vdots\, \pi_2 \\ a \vdash y \end{matrix}}{x \vdash y}
\quad \rightsquigarrow \quad
\begin{matrix} \dfrac{\begin{matrix} \vdots\, \pi_1 \\ x \vdash a \end{matrix} \qquad \begin{matrix} \vdots\, \pi_2' \\ a_u \vdash y' \end{matrix}}{x \vdash y'} \\ \vdots\, \pi_2[x/a] \\ x \vdash y \end{matrix}
$$

CANONICAL CUT ELIMINATION, 4/4

- Subcase (b2): a_u principal in other rule. Then, a_u is in display, and hence:

$$\dfrac{\begin{array}{c}\vdots\;\pi_1\\ x \vdash a\end{array} \qquad \begin{array}{c}\vdots\;\pi_2'\\ a_u \vdash y'\\ \\ \vdots\;\pi_2\\ a \vdash y\end{array}}{x \vdash y} \quad \rightsquigarrow \quad \begin{array}{c}\dfrac{\begin{array}{c}\vdots\;\pi_1\\ x \vdash a\end{array} \qquad \begin{array}{c}\vdots\;\pi_2'\\ a_u \vdash y'\end{array}}{x \vdash y'}\\ \\ \vdots\;\pi_2[x/a]\\ x \vdash y\end{array}$$

- Subcase (b3): a_u parametric. Then:

$$\dfrac{\begin{array}{c}\vdots\;\pi_1\\ x \vdash a\end{array} \qquad \begin{array}{c}\vdots\;\pi_2'\\ (x' \vdash y')[a_u]^{pre}\\ \\ \vdots\;\pi_2\\ a \vdash y\end{array}}{x \vdash y} \quad \rightsquigarrow \quad \begin{array}{c}\vdots\;\pi_2'\\ (x' \vdash y')[x/a_u^{pre}]\\ \\ \vdots\;\pi_2[x/a_u^{pre}]\\ x \vdash y\end{array}$$

CONCLUSIONS & FUTURE WORK

To summarise

✓ Display Calculi ⇝ Multi-type DC ⇝ Display-type Calculi

Current directions

- Linear Logic: avoiding *closed-enough rules*
- PDL: avoiding *omega-rule*, conservativity
- Game Logic
- Display-type Sequent Calculus for Monotonic Modal Logic

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