Algebraic Semantics and Model Completeness for Intuitionistic Public Announcement Logic

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Language

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$$\varphi ::= p \in \mathsf{AtProp} \mid \neg \varphi \mid \varphi \lor \psi \mid \Diamond \varphi \mid \langle \alpha \rangle \varphi.$$

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Axioms

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Language

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Axioms

- $(\alpha \land p) \leftrightarrow (\alpha \land p)$
- $(\alpha \land \neg \varphi \leftrightarrow (\alpha \land \neg \langle \alpha \rangle \varphi)$

$$(\alpha)(\varphi \lor \psi) \leftrightarrow (\langle \alpha \rangle \varphi \lor \langle \alpha \rangle \psi)$$

$$(\alpha \land \Diamond \varphi \leftrightarrow (\alpha \land \Diamond (\alpha \land \langle \alpha \rangle \varphi)).$$

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$$(\alpha)(\varphi \lor \psi) \leftrightarrow (\langle \alpha \rangle \varphi \lor \langle \alpha \rangle \psi)$$

Not amenable to a standard algebraic treatment.

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Semantics of PAL

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 $M, w \Vdash \langle \alpha \rangle \varphi$ iff

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 $M, w \Vdash \langle \alpha \rangle \varphi \qquad \text{iff} \qquad M, w \Vdash \alpha$

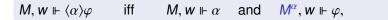
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 $M, w \Vdash \langle \alpha \rangle \varphi$ iff $M, w \Vdash \alpha$ and $M^{\alpha}, w \Vdash \varphi$,

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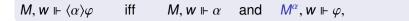
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Relativized model

 $M^{\alpha} = (W^{\alpha}, R^{\alpha}, V^{\alpha})$:

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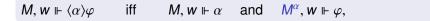


Relativized model

$$M^{\alpha} = (W^{\alpha}, R^{\alpha}, V^{\alpha}):$$

• $W^{\alpha} = \llbracket \alpha \rrbracket_{M},$

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Relativized model

$$M^{\alpha} = (W^{\alpha}, R^{\alpha}, V^{\alpha}):$$

• $W^{\alpha} = \llbracket \alpha \rrbracket_{M},$
• $R^{\alpha} = R \cap (W^{\alpha} \times W^{\alpha}),$

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$$M, w \Vdash \langle \alpha \rangle \varphi \quad \text{iff} \quad M, w \Vdash \alpha \quad \text{and} \quad M^{\alpha}, w \Vdash \varphi,$$

Relativized model

$$M^{\alpha} = (W^{\alpha}, R^{\alpha}, V^{\alpha})$$
:

•
$$W^{\alpha} = \llbracket \alpha \rrbracket_M,$$

•
$$R^{\alpha} = R \cap (W^{\alpha} \times W^{\alpha}),$$

•
$$V^{\alpha}(p) = V(p) \cap W^{\alpha}$$
.

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 Dualize epistemic update on Kripke models to epistemic update on algebras.

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- Dualize epistemic update on Kripke models to epistemic update on algebras.
- Generalize epistemic update on algebras to much wider classes of algebras.
- Dualize back to relational models for non classically based logics.

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An *algebraic model* is a tuple $M = (\mathbb{A}, V)$ s.t. \mathbb{A} is a monadic Heyting algebra and V: AtProp $\rightarrow \mathbb{A}$.

For every \mathbb{A} and every $a \in \mathbb{A}$, define the equivalence relation \equiv_a :

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For every \mathbb{A} and every $a \in \mathbb{A}$, define the equivalence relation \equiv_a : for every $b, c \in \mathbb{A}$,

 $b \equiv_a c$ iff $b \wedge a = c \wedge a$.

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Let $[b]_a$ be the equivalence class of $b \in \mathbb{A}$. Let

$$\mathbb{A}^a := \mathbb{A} / \equiv_a$$

 \mathbb{A}^a is ordered: $[b] \leq [c]$ iff $b' \leq_{\mathbb{A}} c'$ for some $b' \in [b]$ and some $c' \in [c]$.

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 \mathbb{A}^{a} is ordered: $[b] \leq [c]$ iff $b' \leq_{\mathbb{A}} c'$ for some $b' \in [b]$ and some $c' \in [c]$. Let $\pi_{a} : \mathbb{A} \to \mathbb{A}^{a}$ be the canonical projection.

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Properties of the (pseudo)-congruence

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Properties of the (pseudo)-congruence

For every \mathbb{A} and every $a \in \mathbb{A}$,

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• \equiv_a is a congruence if \mathbb{A} is a BA / HA / BDL / Fr.

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Crucial remark

Each \equiv_a -equivalence class has a canonical representant. Hence, the map $i' : \mathbb{A}^a \to \mathbb{A}$ given by $[b] \mapsto b \land a$ is injective.

Modalities of the pseudo-quotient

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Let $(\mathbb{A}, \diamondsuit, \Box)$ be a HAO. Define for every $b \in \mathbb{A}$,

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Let $(\mathbb{A}, \diamond, \Box)$ be a HAO. Define for every $b \in \mathbb{A}$,

$$\diamond^{a}[b] := [\diamond(b \land a) \land a] = [\diamond(b \land a)].$$
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For every HAO $(\mathbb{A}, \diamondsuit, \Box)$ and every $a \in \mathbb{A}$,

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For every HAO $(\mathbb{A}, \diamondsuit, \Box)$ and every $a \in \mathbb{A}$,

- \diamond^a , \Box^a are normal modal operators.
- If $(\mathbb{A}, \diamondsuit, \Box)$ is an MHA, then $(\mathbb{A}^a, \Box^a, \diamondsuit^a)$ is an MHA.

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- \diamond^a , \Box^a are normal modal operators.
- If $(\mathbb{A}, \diamond, \Box)$ is an MHA, then $(\mathbb{A}^a, \Box^a, \diamond^a)$ is an MHA.
- If $\mathbb{A} = \mathcal{F}^+$ for some Kripke frame \mathcal{F} , then $\mathbb{A}^a \cong_{BAO} \mathcal{F}^{a^+}$.

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Let $i: M^{\alpha} \hookrightarrow M$.

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Let $i: M^{\alpha} \hookrightarrow M$. The satisfaction condition

 $M, w \Vdash \langle \alpha \rangle \varphi$ iff $M, w \Vdash \alpha$ and $M^{\alpha}, w \Vdash \varphi$:

can be equivalently written as follows:

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 $w \in \llbracket \langle \alpha \rangle \varphi \rrbracket_M$ iff $\exists w' \in W^{\alpha} \text{ s.t. } i(w') = w \in \llbracket \alpha \rrbracket_M \text{ and } w' \in \llbracket \varphi \rrbracket_{M^{\alpha}}.$

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Because $i: M^{\alpha} \hookrightarrow M$ is injective, then

$$w' \in \llbracket \varphi \rrbracket_{M^{\alpha}}$$
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$$w' \in \llbracket \varphi \rrbracket_{M^{\alpha}}$$
 iff $w = i(w') \in i[\llbracket \varphi \rrbracket_{M^{\alpha}}].$

Hence:

$$w \in \llbracket \langle \alpha \rangle \varphi \rrbracket_M \quad \text{iff} \quad w \in \llbracket \alpha \rrbracket_M \cap i \llbracket \varphi \rrbracket_{M^{\alpha}}],$$

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Hence:

$$w \in \llbracket \langle \alpha \rangle \varphi \rrbracket_M \quad \text{iff} \quad w \in \llbracket \alpha \rrbracket_M \cap i \llbracket \varphi \rrbracket_{M^{\alpha}}],$$

from which we get

$$\llbracket \langle \alpha \rangle \varphi \rrbracket_M = \llbracket \alpha \rrbracket_M \cap i \llbracket \varphi \rrbracket_{M^{\alpha}} = \llbracket \alpha \rrbracket_M \cap i' (\llbracket \varphi \rrbracket_{M^{\alpha}}).$$

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For every algebraic model $M = (\mathbb{A}, V)$, the *extension map* $\llbracket \cdot \rrbracket_M : Fm \to \mathbb{A}$ is defined recursively as follows:

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For every algebraic model $M = (\mathbb{A}, V)$, the *extension map* $\llbracket \cdot \rrbracket_M : Fm \to \mathbb{A}$ is defined recursively as follows:

$$\begin{split} \llbracket p \rrbracket_{M} &= V(p) \\ \llbracket \bot \rrbracket_{M} &= \bot^{\mathbb{A}} \\ \llbracket \top \rrbracket_{M} &= \top^{\mathbb{A}} \\ \llbracket \varphi \lor \psi \rrbracket_{M} &= \llbracket \varphi \rrbracket_{M} \lor^{\mathbb{A}} \llbracket \psi \rrbracket_{M} \\ \llbracket \varphi \land \psi \rrbracket_{M} &= \llbracket \varphi \rrbracket_{M} \land^{\mathbb{A}} \llbracket \psi \rrbracket_{M} \\ \llbracket \varphi \to \psi \rrbracket_{M} &= \llbracket \varphi \rrbracket_{M} \to^{\mathbb{A}} \llbracket \psi \rrbracket_{M} \\ \llbracket \varphi \to \psi \rrbracket_{M} &= \llbracket \varphi \rrbracket_{M} \to^{\mathbb{A}} \llbracket \psi \rrbracket_{M} \\ \llbracket \varphi \to \psi \rrbracket_{M} &= \llbracket \varphi \rrbracket_{M} \to^{\mathbb{A}} \llbracket \psi \rrbracket_{M} \\ \llbracket \varphi \varphi \rrbracket_{M} &= \square^{\mathbb{A}} \llbracket \varphi \rrbracket_{M} \\ \llbracket \Box \varphi \rrbracket_{M} &= \square^{\mathbb{A}} \llbracket \varphi \rrbracket_{M} \\ \llbracket [\Box \varphi \rrbracket_{M} &= \llbracket \alpha \rrbracket_{M} \land^{\mathbb{A}} i' (\llbracket \varphi \rrbracket_{M}^{\alpha} \\ \llbracket [\alpha] \varphi \rrbracket_{M} &= \llbracket \alpha \rrbracket_{M} \to^{\mathbb{A}} i' (\llbracket \varphi \rrbracket_{M}^{\alpha} \end{split}$$

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 $M^{\alpha} := (\mathbb{A}^{\alpha}, V^{\alpha}) \text{ s.t. } \mathbb{A}^{\alpha} = \mathbb{A}^{\llbracket \alpha \rrbracket_{M}} \text{ and } V^{\alpha} : \text{AtProp} \to \mathbb{A}^{\alpha} \text{ is } \pi \circ V, \text{ i.e.}$ $\llbracket p \rrbracket_{M^{\alpha}} = V^{\alpha}(p) = \pi(V(p)) = \pi(\llbracket p \rrbracket_{M}) \text{ for every } p.$

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Intuitionistic PAL

Minghui Ma, Alessandra Palmigiano, Mehrnoosh Sadrzadeh Algebraic Semantics and Model Completeness for Intuitionistic Public

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Intuitionistic PAL

$\varphi ::= p \in \mathsf{AtProp} \mid \bot \mid \top \mid \varphi \lor \psi \mid \varphi \land \psi \mid \varphi \to \psi \mid \Diamond \varphi \mid \Box \varphi \mid \langle \alpha \rangle \varphi \mid [\alpha] \varphi.$

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 $[\alpha] \top = \top$

Interaction with disjunction

 $\langle \alpha \rangle (\varphi \lor \psi) = \langle \alpha \rangle \varphi \lor \langle \alpha \rangle \psi$ $[\alpha](\varphi \lor \psi) = \alpha \to (\langle \alpha \rangle \varphi \lor \langle \alpha \rangle \psi)$

Interaction with implication

 $\langle \alpha \rangle (\varphi \to \psi) = \alpha \land (\langle \alpha \rangle \varphi \to \langle \alpha \rangle \psi)$ $[\alpha](\varphi \to \psi) = \langle \alpha \rangle \varphi \to \langle \alpha \rangle \psi$

Interaction with \diamond

 $\langle \alpha \rangle \diamondsuit \varphi = \alpha \land \diamondsuit \langle \alpha \rangle \varphi$ $[\alpha] \diamondsuit \varphi = \alpha \to \diamondsuit \langle \alpha \rangle \varphi$ Interaction with $\langle \alpha \rangle \Box \varphi = \alpha \land \Box [\alpha] \varphi$ $[\alpha]\Box\varphi = \alpha \to \Box[\alpha]\varphi$

Interaction with conjunction

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 $\langle \alpha \rangle (\varphi \land \psi) = \langle \alpha \rangle \varphi \land \langle \alpha \rangle \psi$ $[\alpha](\varphi \land \psi) = [\alpha]\varphi \land [\alpha]\psi$

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 $(R \circ \geq) \subseteq (\geq \circ R)$ $(\leq \circ R) \subseteq (R \circ \leq)$ $R = (\geq \circ R) \cap (R \circ \leq);$

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- **Epistemic updates** defined exactly in the **same** way as in the Boolean case.

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Work in progress: Intuitionistic account of Muddy Children Puzzle.