Noncommutative spaces and their representation theory

Alessandra Palmigiano
(joint work with Riccardo Re)

TACL, 28 July 2011

Stone duality

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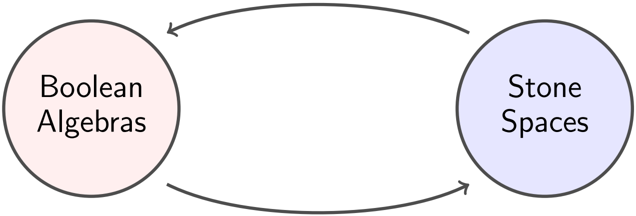

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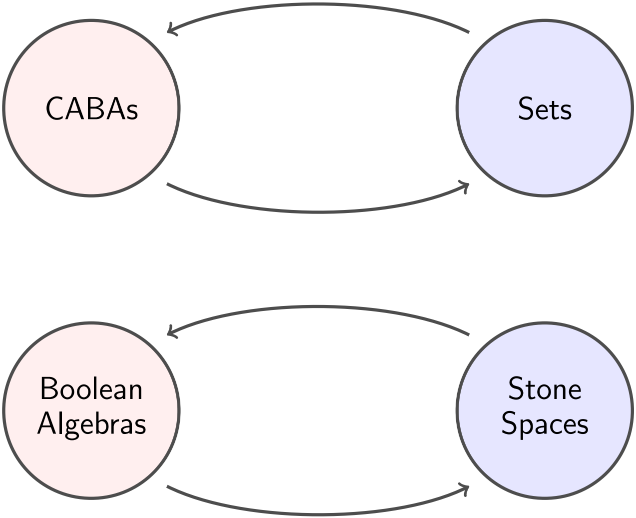

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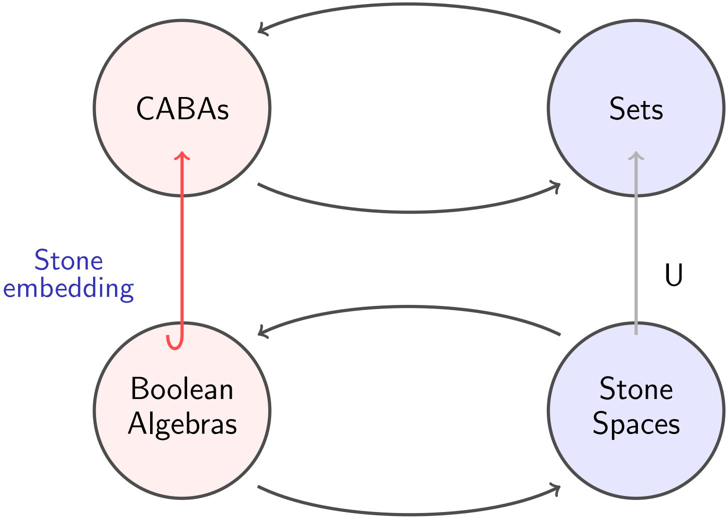

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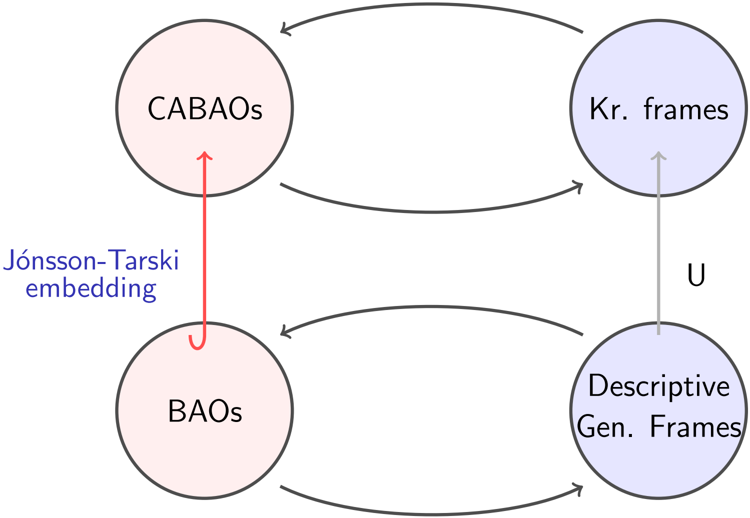

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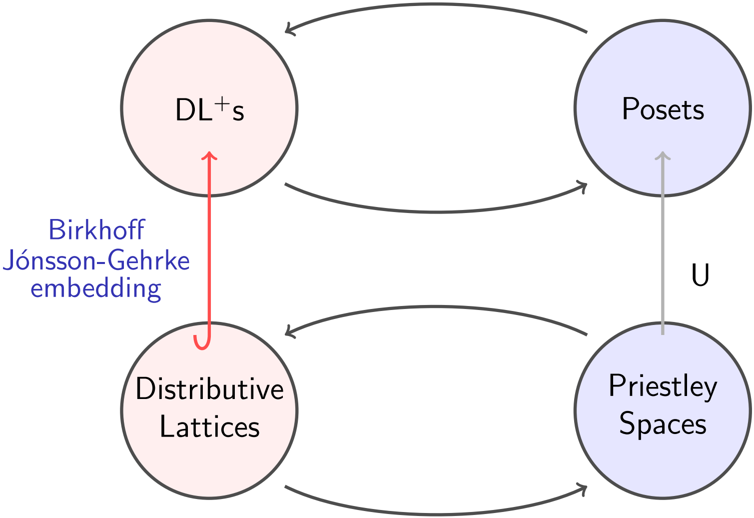

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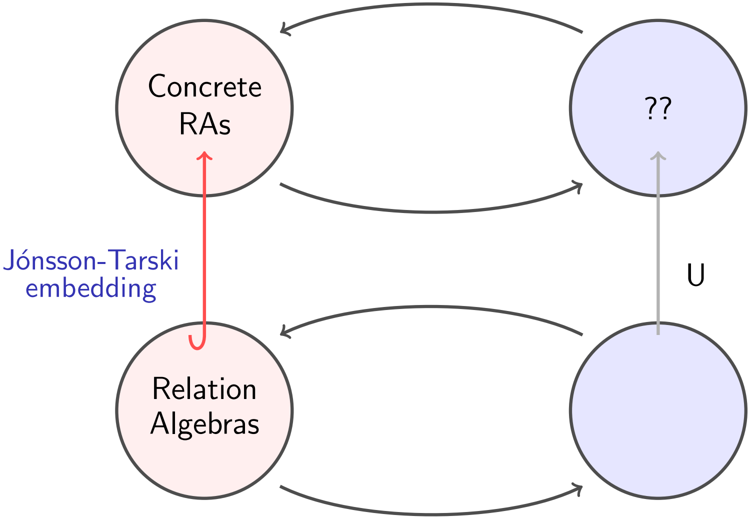

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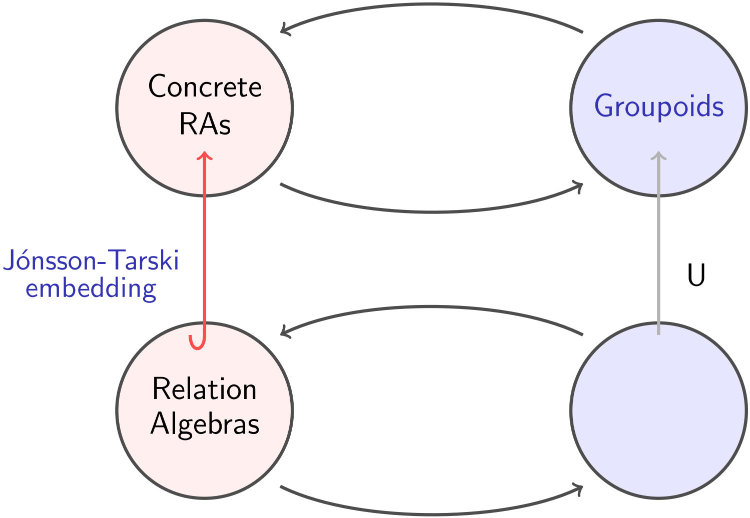

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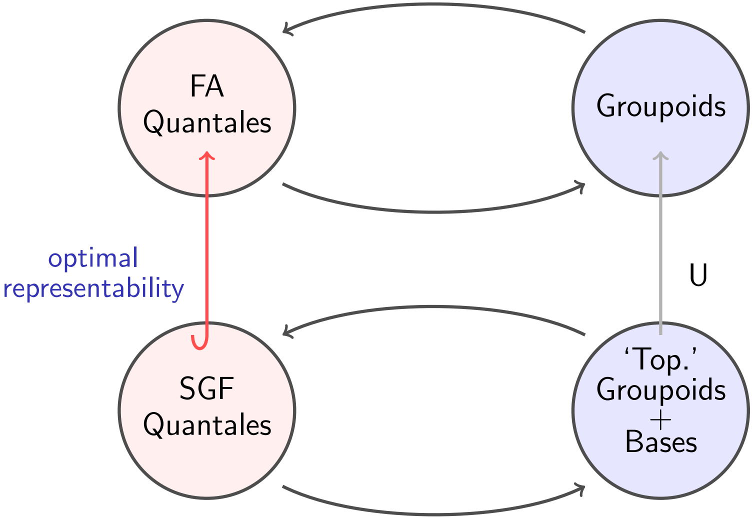

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- important instances mediated by (étale) groupoids;
- groupoids as ‘noncommutative spaces’.

- Quantales as models of a geometric logic of binary relations;
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- Relations as groupoids are not naturally ‘étale’.

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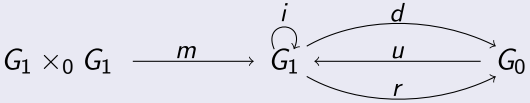
Set groupoids are tuples

$$G = (G_0, G_1, m, d, r, u, i)$$

s.t. G_0 and G_1 are sets, and:



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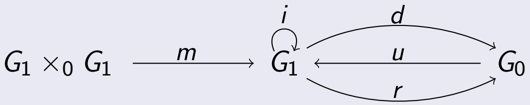

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Topological/Localic Groupoids: internal groupoids in Top/Loc.
Étale Groupoids: structure map d is a local homeomorphism.

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Fact: Bisection images naturally form a (unital) inverse semigroup:
- the unit is $u[G_0]$;
- product is composition.

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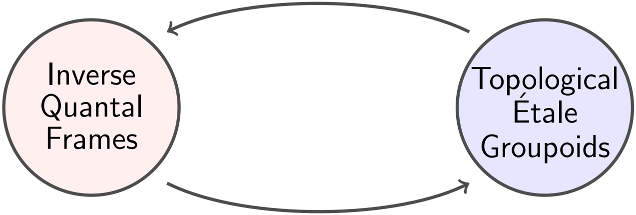

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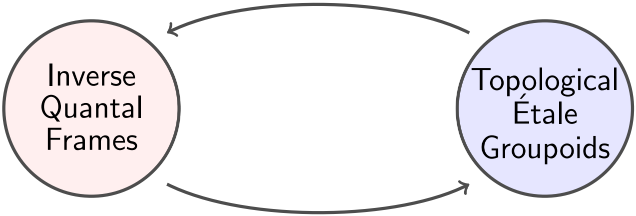

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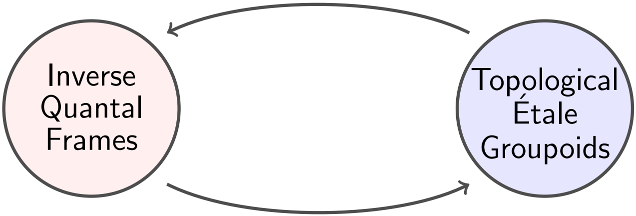

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$a \in Q$ s.t. $aa^* \leq e$ and $a^*a \leq e$
(Jónsson-Tarski's functional invertible elements)

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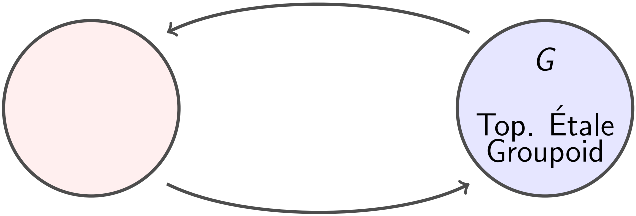

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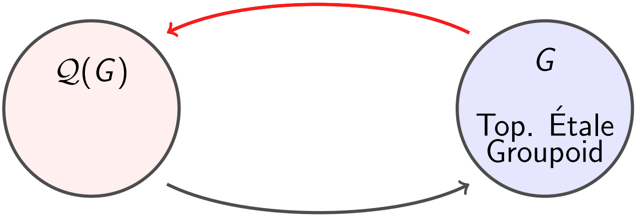

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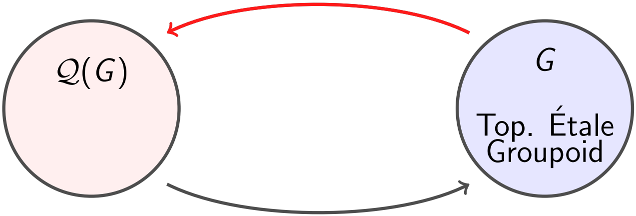

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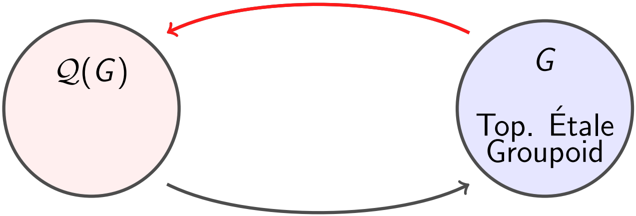

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Bisection images of G $\leftrightsquigarrow$ partial units of $\mathcal{Q}(G)$

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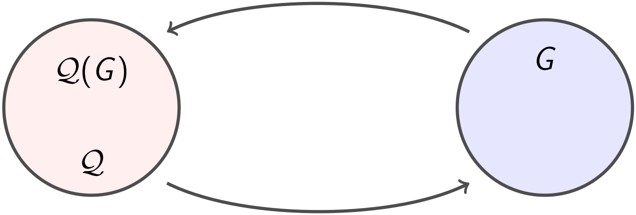

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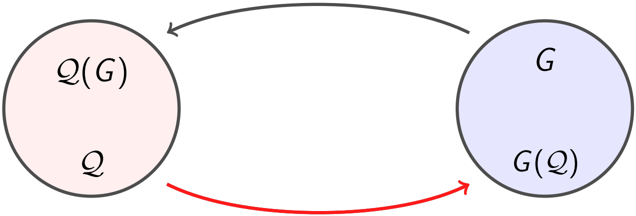

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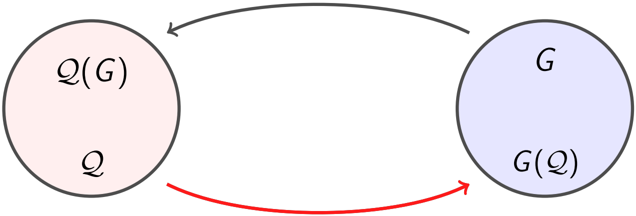

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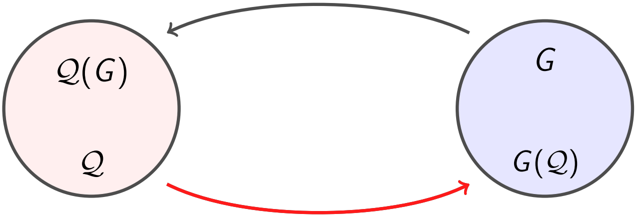

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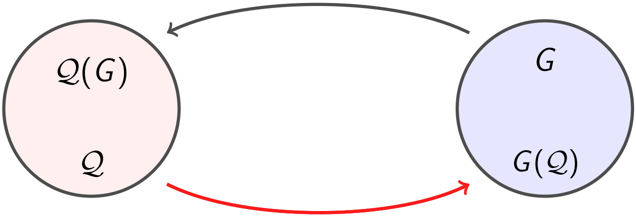

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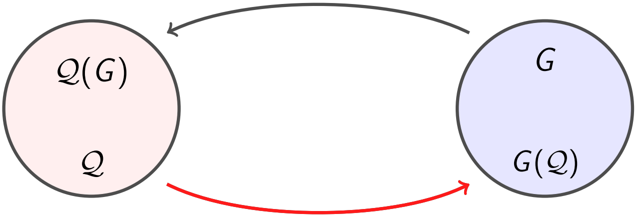

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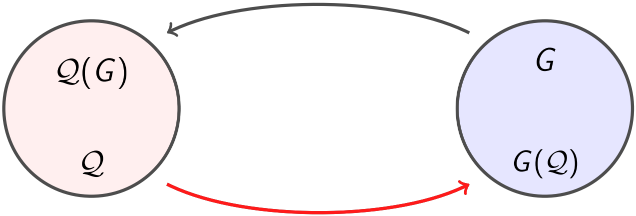

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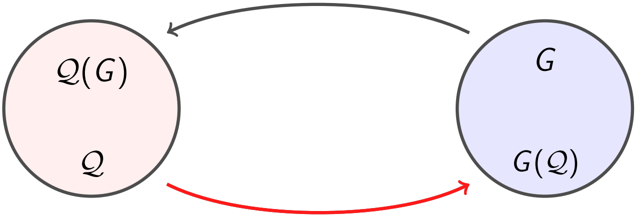

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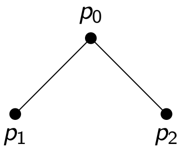
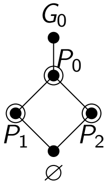
Bisect. images form a **base** for the top. $\mathcal{Q}$

A groupoid whose bis. im's are not a base for any top.

Let $X = (G_0, \Omega(G_0))$ (topology $\Omega(G_0)$ given by down-sets):





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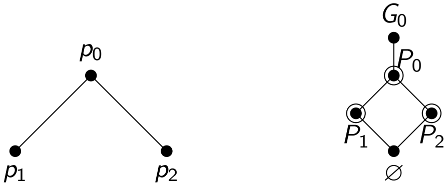

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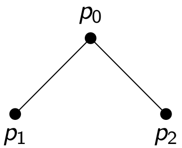

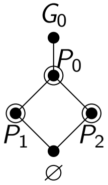

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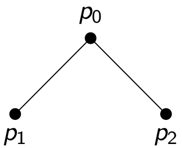

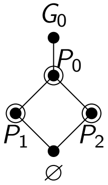

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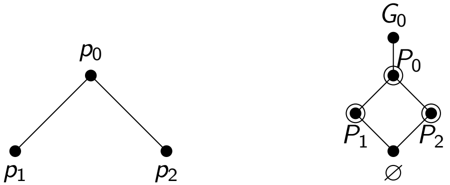

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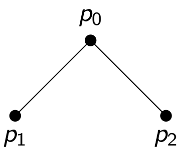

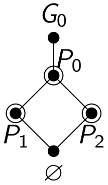

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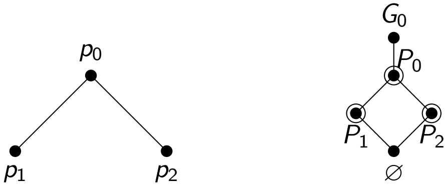

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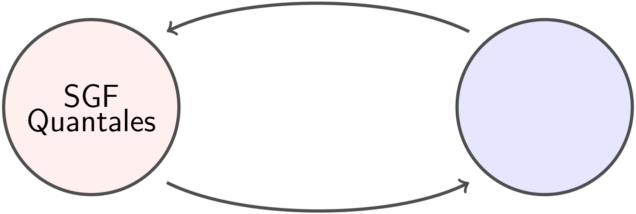

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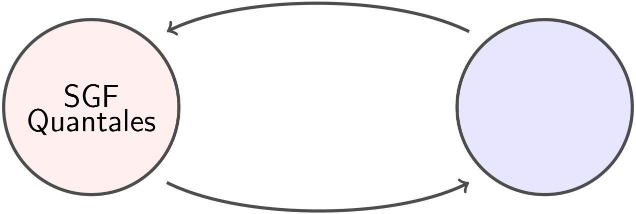

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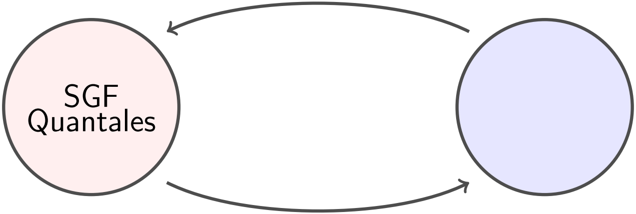

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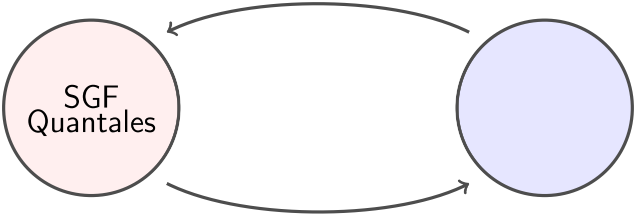

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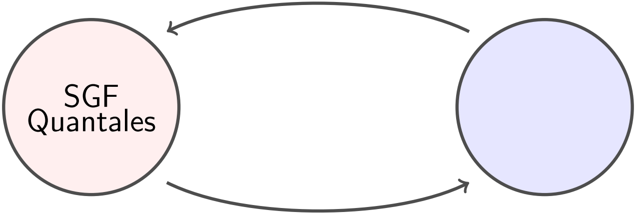

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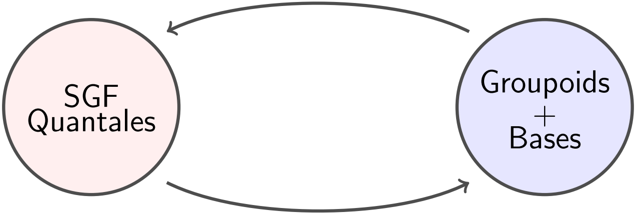

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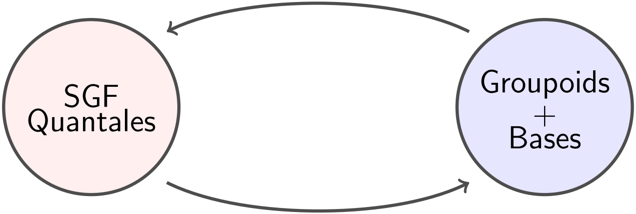

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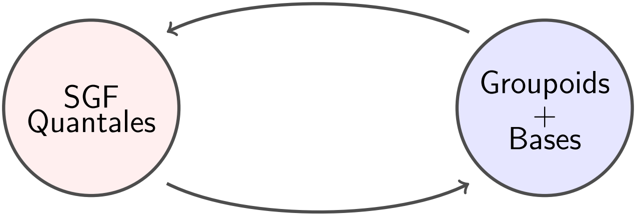

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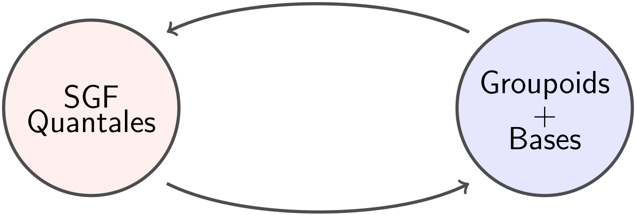
The non étale duality (on objects) [P. - Re 2011a]



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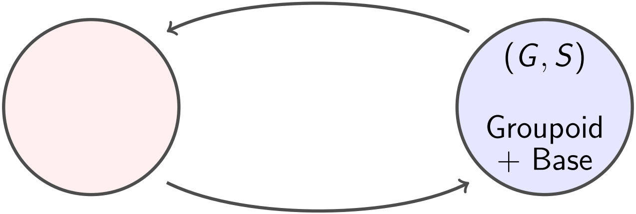

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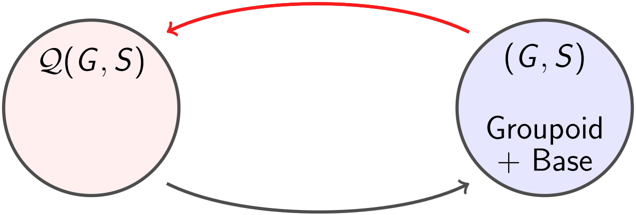

defined as before, except
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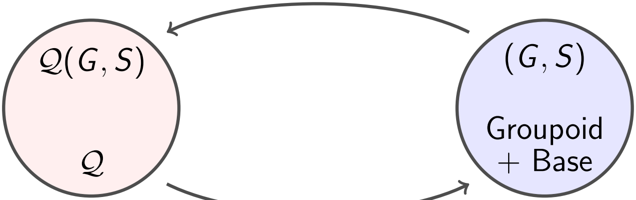

SGF
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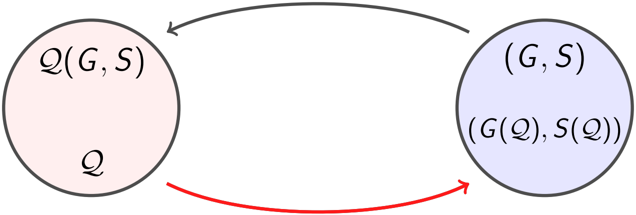

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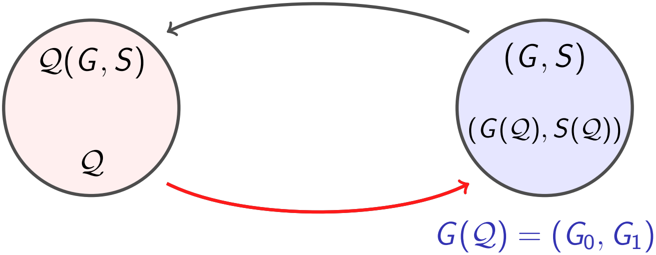

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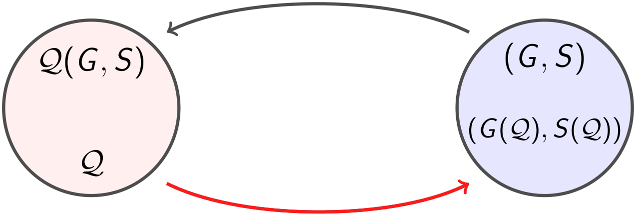

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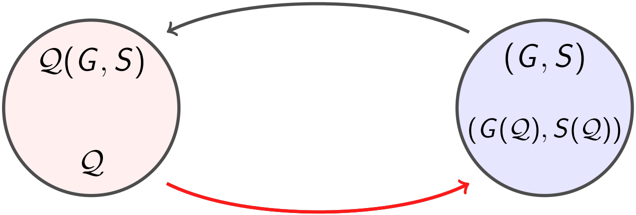

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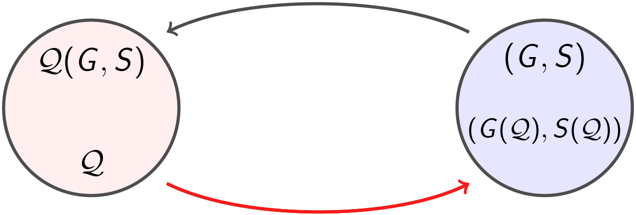

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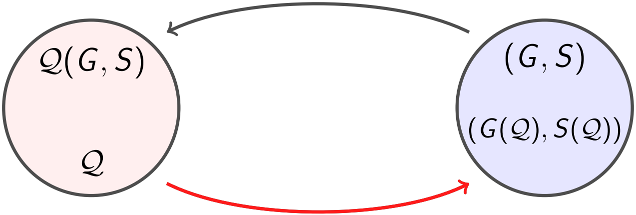

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$$d([p, f]) = p, \quad r([p, f]) = f[p], \quad u(p) = [p, e],$$

$$[p, f][q, g] = [p, fg] \quad \text{only if} \quad q = f[p]$$

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- $\mathcal{S}$ covers G_1.