A Generalization of Topology with an Eye on Stone Duality

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Motivation

Stone Duality for Bitopological Spaces

Recent joint work with Achim Jung:

- Unifies several Stone-type dualities in a bitopological setting.
- Replaces two element lattice by Belnap's four element bilattice with additional structure.
- Exploits an interesting distinction between "logic" and "information."

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This Talk

- Considers other generalizations of topology via other dualizing objects.
- Illustrates the value of maintaining "logic" versus "information."

Ingredients of point-set topology

The Familiar Definition

- A topology on a set X is a family $\tau \subseteq \mathcal{P}(X)$ closed under finite intersection and arbitrary union.
- A continuous function from (X, σ) to (Y, τ) is a function from X to Y so that $f^{-1}(V) \in \sigma$ for each $V \in \tau$.

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Alternate Definition

- A topology on a set X is a sub-frame τ of 2^X.
- A continuous function from (X, σ) to (Y, τ) is a function from X to Y so that $v \circ f \in \sigma$ for each $v \in \tau$.

Recalling Frames (with apologies for confusion with Kripke frames)

Definition

- A frame is a complete lattice satisfying the frame law:

$$a \wedge \bigvee B = \bigvee_{b \in B} (a \wedge b)$$

- A frame homomorphism preserves $(\top, \wedge, \bot, \bigvee)$.

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Alternate Definition

- A frame is
  - a distributive lattice;
  - a dcpo in its lattice order;
  - having Scott continuous meet (and join).
- A frame homomorphism is a Scott continuous distributive lattice homomorphism.

The Neighborhood Map

The characteristic of open neighborhoods of a point:

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Theorem

The contravariant hom-set functors $\mathbf{Top}(-, \mathbb{S})$ and $\mathbf{Frm}(-, \mathbf{2})$ are interpretable as adjoint functors $\Omega \colon \mathbf{Top} \to \mathbf{Frm}^{\mathrm{op}}$ and $\mathrm{spec} \colon \mathbf{Frm} \to \mathbf{Top}^{\mathrm{op}}$.

Bitopology

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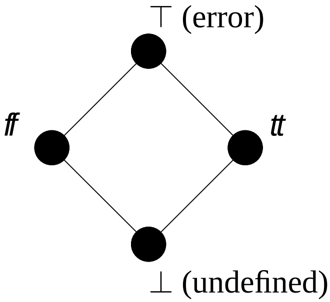
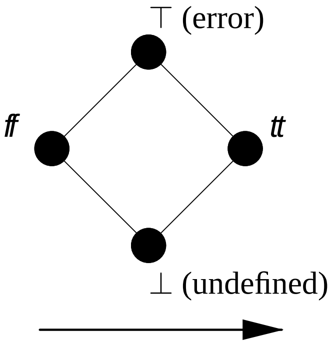
- $\mathbb{R}$ with the upper open topology and the lower open topology.
- A sober space with its given topology and its co-compact topology.
- If $\leq \subseteq X \times X$ is a topologically closed partial order on X, then the upper open and lower open sets form a bitopology (generalizes $\mathbb{R}$).

An alternate “Truth value” lattice (2.2 – Belnap’s lattice)





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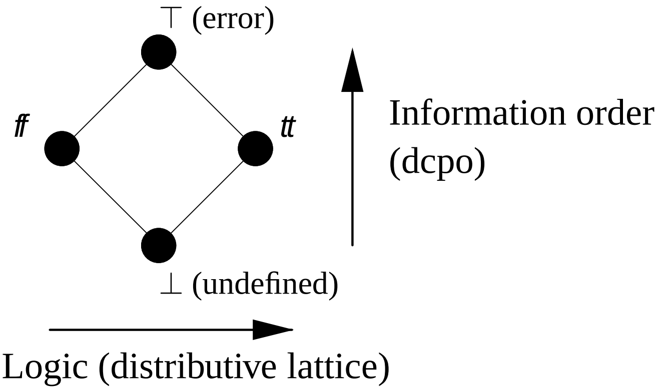

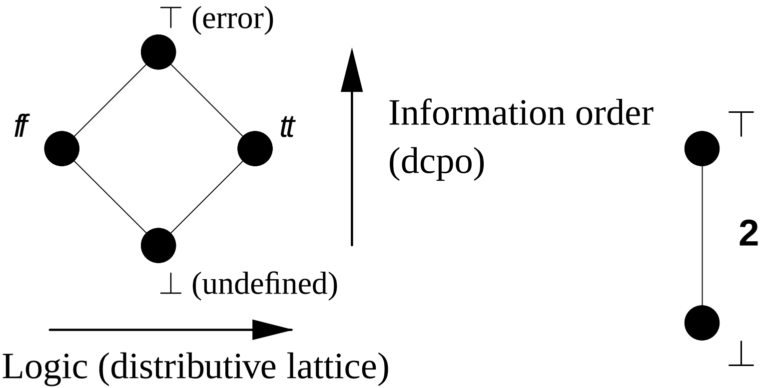
⊤ (error)
ff
tt
⊥ (undefined)
Logic (distributive lattice)
Information order
(dcpo)
⊤
2
⊥

Alternate Definition

- A bitopology on a set X is a collection $\tau \subseteq (\mathbf{2.2})^X$ so that
  - τ is closed under $\wedge$ and $\vee$;
  - τ is closed under suprema of directed sets;
  - τ includes all (four) constant functions.

  The operations are defined pointwise.
- A bicontinuous function from (X, σ) to (Y, τ) is a function f from X to Y so that $u \circ f \in \sigma$ for each $u \in \tau$.

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Lemma

The standard and alternate definitions of bitopologies and bicontinuity are equivalent.

What about the adjunction $\Omega \dashv \mathrm{spec}$?

Definition

Let $\mathbb{S}.\mathbb{S}$ denote the bitopology on the underlying set $\mathbf{2}.\mathbf{2}$ equipped with the bitopology generated by $\mathrm{id}\colon \mathbf{2}.\mathbf{2} \to \mathbf{2}.\mathbf{2}$.

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Definition

- A trestle is a structure $\mathbf{L} = (L; \wedge, tt, \vee, ff; \sqsubseteq, \bot)$ so that
  - $(L; \wedge, tt, \vee, ff)$ is a bounded distributive lattice;
  - $(L; \sqsubseteq, \bot)$ is a dcpo with least element $\bot$;
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- A trestle homomorphism preserves all of this structure.

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Lemma

For any bitopological space $\mathcal{X} = (X, \tau)$, the trestle τ is isomorphic to $\mathbf{biTop}(\mathcal{X}, \mathbb{S}.\mathbb{S})$ where the operations are defined point-wise.

Lemma

For two frames K and L, impose a bitopology on $\mathbf{Frm}^2(K \times L, \mathbf{2.2})$ generated by:

$$U_(a, b)(h) := h(a, b)$$

The maps U_u form a bitopology. In particular, $u \mapsto U_u$ is a surjective homomorphism in $\mathbf{Frm}^2$. So $\mathbf{Frm}^2(-, \mathbf{2.2})$ determines a contravariant functor $\mathrm{spec} \colon \mathbf{Frm}^2 \to \mathbf{biTop}$

Bitopological $\Omega \dashv \mathrm{spec}$

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Theorem

The functors Ω and spec are dually adjoint.
[N.B. $\mathbf{Frm}^2$ is a full subcategory of **Tre**, as is **Frm**.]

Generalized Topology

Definition

Let **T** be any fixed trestle.

- A **T**-topology on set X is a sub-trestle $\tau \subseteq \mathbf{T}^X$ that includes all constant functions: $x \mapsto a$ for each $a \in \mathbf{T}$.
- A **T**-space is a set equipped with a **T**-topology.
- A **T**-continuous map from (X, σ) to (Y, τ) is a map from X to Y so that $u \circ f \in \sigma$ for each $u \in \tau$.

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- **2**-topologies are topologies; **2**-continuous functions are continuous functions.
- **2**.**2**-topologies are bitopologies; **2**.**2**-continuous functions are bicontinuous functions.
- **1**-topologies are sets; **1**-continuous functions are functions.

The functor $\Omega_{\mathbf{T}}$

Lemma

For any **T**-continuous function $f\colon (X, \sigma) \to (Y, \tau)$, the map $v \mapsto v \circ f$ is a trestle homomorphism from τ to σ.

The functor $\Omega_{\mathbf{T}}$

Lemma

For any **T**-continuous function $f: (X, \sigma) \to (Y, \tau)$, the map $v \mapsto v \circ f$ is a trestle homomorphism from τ to σ.

Definition

$\Omega_{\mathbf{T}}(X, \tau) := \tau$ and $\Omega_{\mathbf{T}}(f) := (v \mapsto v \circ f)$ define a contravariant functor from **T-space** to **Tres**.

- $\Omega_{\mathbf{2}}(f) = f^{-1}$ restricted to open sets.
- $\Omega_{\mathbf{1}}(f) = f^{-1}$ unrestricted.
- $\Omega_{\mathbf{2}.\mathbf{2}}(f)$ is determined by f^{-1} restricted to opens in the two underlying topologies.

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Proof Sketch

Define $\tau_{\mathbf{T}}$ to be the **T**-topology on (the underlying set of) **T** generated by $\mathrm{id}\colon \mathbf{T} \to \mathbf{T}$.

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So $\Omega_{\mathbf{T}}$ is represented by the hom-set functor **T-space**$(-, \mathbf{T})$.

Proof Sketch continued

Define $\text{spec}_{\mathbf{T}} : \mathbf{Tres} \to \mathbf{T\text{-}space}$ by

- $\text{spec}_{\mathbf{T}}(\mathbf{L}) := \mathbf{Tres}(\mathbf{L}, \mathbf{T})$.
- The $\mathbf{T}$-topology is generated by the functions $B_u : \text{spec}_{\mathbf{T}}(\mathbf{L}) \to \mathbf{T}$

  $$B_u(p) := p(u)$$

  for each $u \in \mathbf{L}$.
- $\text{spec}_{\mathbf{T}}(h)(p) = p \circ h$

Basic Theorem continued

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Simple definition chasing shows that

$$\textbf{Tres}(\mathbf{L}, \Omega_{\mathbf{T}}(\mathcal{X})) \simeq \textbf{T-space}(\mathcal{X}, \text{spec}_{\mathbf{T}}(\mathbf{L}))$$

naturally in $\mathbf{L}$ and $\mathcal{X}$. □

Sobriety

Unit

On the “spatial side”, the unit of the adjunction $\eta\colon \mathfrak{X} \to \mathrm{spec}_{\mathbf{T}}(\Omega_{\mathbf{T}}(\mathfrak{X}))$ is given by

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For **T**-topology $\mathcal{X}$, the following are equivalent:

- $\mathcal{X} \simeq \mathrm{spec}_{\mathbf{T}}(\Omega_{\mathbf{T}}(\mathcal{X}))$
- η is an isomorphism
- η is a bijection
- η is a surjection and $\mathcal{X} \simeq \mathrm{spec}_{\mathbf{T}}(\mathbf{L})$ for some **L**.

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Definition

A **T**-space is sober iff it satisfies these conditions.

Spatiality

Co-unit

On the "algebra side", the (co)unit of the adjunction $\epsilon\colon \mathbf{L} \to \Omega_{\mathbf{T}}\mathrm{spec}_{\mathcal{T}}$ is given by

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For a trestle **L**, the following are equivalent:

- $\mathbf{L} \simeq \Omega_{\mathbf{T}}(\mathrm{spec}_{\mathbf{T}}(\mathbf{L}))$
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Definition

A trestle is **T**-spatial iff it satisfies these conditions.

Equivalence

Theorem

If **T** is itself **T**-spatial, then the functors $\Omega_{\mathbf{T}}$ and $\mathrm{spec}^*_{\mathbf{T}} := \mathrm{spec}_{\mathbf{T}} \circ \Omega_{\mathbf{T}} \circ \mathrm{spec}_{\mathbf{T}}$ cut down to a dual equivalence between the categories of sober **T**-spaces and **T**-spatial trestles.

Proof

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Proof

Let $T := \Omega_{\mathbf{T}} \circ \mathrm{spec}_{\mathbf{T}}$ and $S := \mathrm{spec}_{\mathbf{T}} \circ \Omega_{\mathbf{T}}$. Then

- $T(\mathbf{L})$ is **T**-spatial
- $S(\mathcal{X})$ is **T**-sober
- $\Omega_{\mathbf{T}}(\mathcal{X})$ is **T**-spatial
- $\mathrm{spec}^*_{\mathbf{T}} = S \circ \mathrm{spec}_{\mathbf{T}}$, so $\mathrm{spec}^*_{\mathbf{T}}(\mathbf{L})$ is **T**-sober.

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Thus

- If $\mathcal{X} \simeq S(\mathcal{X})$, then $\mathcal{X} \simeq S^2(\mathcal{X}) = \mathrm{spec}^*_{\mathbf{T}}(\Omega_{\mathbf{T}}(\mathcal{X}))$
- If $\mathbf{L} \simeq T(\mathbf{L})$ then $\mathbf{L} \simeq T^2(\mathbf{L}) = \Omega_{\mathbf{T}}(\mathrm{spec}^*_{\mathbf{T}}(\mathbf{L}))$

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$$\sigma \otimes \tau(x, y) := (\sigma(x), \tau(y))$$

- A **2**.**2**-spatial trestle is a product of two spatial frames $K \times L$ with $\sqsubseteq$ being the frame order, $(a, b) \leq (a', b')$ holding if and only if $a \leq a'$ and $b \geq b'$

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Question

In these examples, the equivalency theorem strengthens because $\mathrm{spec}_{\mathbf{T}}(\mathbf{L})$ is already sober.

What characterizes trestles for which this is true?

Example: V-spaces (entangled topology)

Entanglement of the two classical truth values

Let **V** be the set $[0, 1]$ with $\leq$ as the lattice order and $\frac{1}{2}$ as the least element in the information order. [Keye Martin's Bayesian order.]

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- An "open" $u \in \tau$ assigns an 'entanglement' of $\{0, 1\}$ to each $x \in X$.
- More information means "more certainly 0 or more certainly 1."
- Think of the map $u \mapsto u(x)$ as characterizing the state of a "particle" x.

V-spaces continued

V-sobriety

(X, τ) is **V**-sober if and only if

- For each $x \neq y$, there exists $u \in \tau$ so that $u(x) \neq u(y)$.
- For any trestle map such that $h(\kappa_q) = q$ for all $q \in [0, 1]$, there is an x in that state: $h(u) = u(x)$.

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I do not have an internal characterization of **V**-spatial trestles yet.

V is not **V**-spatial, so the theorem that allows us to cut down to an equivalence does not apply.

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Conclusions

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- At least some of these already occur naturally (topology, bi-topology, fuzzy topology).
- The concepts of T_0 separation, sobriety and spatiality generalize to yield a form of Stone duality relative to any suitable concept truth values.
- The key idea is the allow logic and information to determine separate orders.
- The main open questions:
  - Which trestles **T** are already **T**-spatial (thus cutting the adjunction down to an equivalence)?
  - Are there principles to allow for added structure on trestles to obtain more interesting spatial categories?
  - Are there other (better) ways of thinking about the interplay between logic and information?