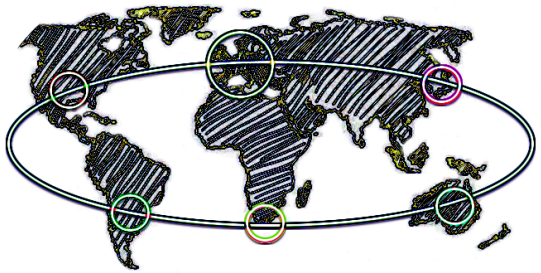
Scientific Work Packages

Mid-term meeting

SYSMICS project

http://logica.dmi.unisa.it/sysmics/

University of Salerno



WP 4

Abstract Algebraic Logic

Involved groups

- Institute of Computer Science (Czech Republic)
- Institute of Information Theory and Automation (Czech Republic)
- University of Barcelona (Spain)
- University of Witwatersrand (South Africa)
- CONICET (Argentina)
- University of Denver Colorado (United States)
- La Trobe University (Australia)

Overall goal of the WP and its tasks

The global objective of this Work Package is to advance the state of the art in Abstract Algebraic logic.
The WP has the following tasks:

1. Equivalence between logics in different languages.
2. Equationally orderable quasivarieties and their associated sequent calculi and deductive systems.
3. Relations between deduction theorems, proof by cases and filter generation.
4. Beth's definability property.
5. Uniform framework for non-classical predicate logics and their model theory.
6. Consequence relations with multisets of premises and conclusions.

A representative result for each task ...

Task 1:

T. Moraschini.
A logical and algebraic characterisation of adjunctions between generalised quasi-varieties. Submitted.

In this paper we describe a correspondence between logical translations and adjunctions between quasi-varieties. A combinatorial canonical form for right adjoint functors is obtained, which generalises McKenzie's description of category equivalences

Task 2:

R. Jansana.
On the Deductive System of the Order of an Equationally Orderable Quasivariety. Stud. Log. 104:547–566, 2016.

This paper investigates deductive systems associated to quasi-varieties, which carry an equationally definable partial order, within the formalism of Gentzen systems.

A representative result for each task ...

Task 3:

P. Cintula, C. Noguera.
Implicational (semilinear) logics III: completeness properties.
To appear in Arch. Math. logic.

In this paper we have used the proof by case property to prove new strong general completeness results.

Task 4:

G. Bezhanishvili, T. Moraschini, J. Raftery.
Epimorphisms and Beth definability. To appear in Journal of Algebra.

In this paper we show that epimorphisms are surjective in all varieties of Heyting algebras of finite height and in all varieties of Sugihara monoids. Moreover, we present the first example of a variety where almost-onto epimorphisms are suprjective, but arbitrary epimorphisms may fail to be surjective.

A representative result for each task ...

Task 5:

P. Dellunde, A. Garcia-Cerdana, C. Noguera.
Back-and-forth systems for first-order fuzzy logics. Submitted.

In this paper we have studied elementary equivalence in the framework of fuzzy and substructural logics. We show that the classical notion splits into three different notions of elementary equivalence in the case of fuzzy models. The main result is a back and forth sufficient conditions that ensure that two fuzzy models are elementarily equivalent.

Task 6:

P. Cintula, J. Gil Férez, T. Moraschini, F. Paoli.
An Abstract Approach to Consequence Relations. Submitted.

In this very extensive paper we lay down abstract foundations for multiset-based consequence relation.

WP 5

Analytic Calculi

Involved groups

- Vienna University of Technology (Austria)
- University of Milan (Italy)
- University of Bern (Switzerland)
- Kyoto University (Japan)
- The Australian National University (Australia)
- Nanyang Technological University (Singapore)

Overall goal of the WP and its tasks

The global objective of this Work Package is to advance the state of the art in Analytic Proof Theory for substructural logics.
The WP has the following tasks:

1.1 Embedding hypersequents into systems of rules.

1.2 Curry-Howard correspondence for Gödel logic.

2.1 Bunched hypersequent calculi for distributive substructural logics.

2.2 Algebraic proof theory.

2.3 proof theory for substructural logics with modalities

Embedding hypersequents into systems of rules

Task 1.1:

A. Ciabattoni, F. A. Genco.
Embedding formalisms: hypersequents and two-level systems of rules. In *AIML 2016*, pp. 197–216.

Hypersequents are a simple and local generalisation of sequents that can be used to study the proof theory of large classes.
Systems of rules are a very expressive and quite complex proof-theoretical formalism based on non-local mechanisms In this paper we define a bi-directional embedding between hypersequents and a subclass of systems of rules (2-systems).
The embedding shows that the two frameworks have the same expressive power and can be used to

- show analyticity results for a large class of 2-systems.
- rewrite hypersequent rules as natural deduction rules.

Curry-Howard correspondence for Gödel logic

Task 1.2:

F. Aschieri, A. Ciabattoni, F. A. Genco.
Curry–Howard correspondence for Gödel logic: from natural deduction to parallel computation. To appear in *LICS 2017*, Reykjavik.

A. Ciabattoni, F. A. Genco.
Hypersequents and systems of rules: embeddings and Applications. Submitted.

We define a simple natural deduction calculus NG from HG, we give a Curry–Howard correspondence for G based on NG and we introduce the system λ_G that extends simply typed λ-calculus by a synchronous communication mechanism between parallel processes. The calculus λ_G is strictly more expressive than simply typed λ-calculus. The normalisation of λ_G exploits original termination arguments and proof transformations implementing forms of code mobility.

Bunched hypersequent calculi for distributive substructural logics

Task 2.1:

A. Ciabattoni, R. Ramanayake.
Bunched Hypersequent Calculi for Distributive Substructural Logics. In Proc: LPAR-21 (2017)

Distributive substructural logics extend Full Lambek logic FL with distributivity $A \wedge (B \vee C) \rightarrow (A \wedge B) \vee (A \wedge C)$, while
Bunched calculi provide cutfree proof calculi for extensions of FL with distributivity.
We combine bunched calculi with hypersequents to give cutfree bunched hypersequent calculi for a large class of axiomatic extensions of distributive FL.
This allows us to extend analytic calculi for substructural logics to distributive case.

Algebraic proof theory

Task 2.2:

A. Ciabattoni, N. Galatos, K. Terui.
Algebraic proof theory: hypersequents and hypercompletions.
Annals of Pure and Applied Logic, 168(3): 693-737, 2017

We continue our program of establishing connections between proof-theoretic and order-algebraic properties in the setting of substructural logics.

We provide algebraic foundations for substructural hypersequent calculi and an algorithm to transform P3 axioms into equivalent structural hypersequent rules. Using residuated hyperframes we link strong analyticity in the resulting calculi with a new algebraic completion, which we call hyper-MacNeille completion.

Task 2.3:

B. Lellmann, C. Olarte, E. Pimentel.
A uniform framework for substructural logics with modalities.
T. Eiter and D. Sands (Eds.): LPAR-21. (2017)

In this work we investigate a local system for linear logic based on linear nested sequents. Relying on that system, we propose a general framework for modularly describing systems combining, coherently, substructural behaviors inherited from linear logic and simply dependent multimodalities. This class of systems includes linear, elementary, affine, bounded and subexponential linear logics and extensions of multiplicative additive linear logic with normal modalities, as well as general combinations of them.

WP 6

Structure Theory

Involved groups

- University of Cagliari (Italy)
- University of Salerno (Italy)
- University of Milan (Italy)
- University of Insubria (Italy)
- Olomuc University (Czech Republic)
- University of Witwatersrand (South Africa)
- CONICET (Argentina)
- Vanderbilt University (United States)
- Latrobe University (Australia)

Overall goal of the WP and its tasks

The global objective of this Work Package is to advance the state of the art in Structure theory of residuated lattices.
The WP has the following tasks:

4.1 Study existence and uniqueness of laterally complete, projectable and strongly projectable hulls of e-cyclic semilinear residuated lattices.

4.2 Study the properties of these hulls.

4.3 Investigate the existence and uniqueness of minimal extensions of a GMV algebra, which are laterally complete, projectable, strongly projectable, or orthocomplete.

Conrad Program refers to Paul Conrad's approach to the study of ℓ-groups that analyses the structure of individual or classes of ℓ-groups by primarily focusing on their lattices of convex ℓ-subgroups.

Our work expands Conrad Program to the vastly more general framework of e-cyclic residuated lattices. This variety encompasses most varieties of notable significance in algebraic logic, including ℓ-groups, MV algebras, pseudo-MV algebras, GMV algebras, semilinear GBL algebras, BL algebras, Heyting algebras, commutative residuated lattices, and integral residuated lattices.

J. Gil Férez, A. Ledda, C. Tsinakis
Hulls of Ordered Algebras: Projectability, Strong Projectability and Lateral Completeness, Journal of Algebra, 483, pp. 429-474.

Task 4.1: A pivotal result obtained in our research is the following:

Theorem
Any algebra $\mathbf{L}$ *in a variety* $\mathcal{V}$ *of e-cyclic semilinear residuated lattices is densely embeddable in a laterally complete member of* $\mathcal{V}$*, called* $\mathcal{O}(\mathbf{L})$*.*

Task 4.2: Then, we advance our study of $\mathcal{O}(\mathbf{L})$ by proving that $\mathcal{O}(\mathbf{L})$ is also strongly projectable. In fact, we prove more:

Theorem
Let $\mathbf{L}$ *be an e-cyclic semilinear residuated lattice. Then* $\mathcal{O}(\mathbf{L})$ *is strongly projectable.*

Hence, we have the following consequence:

Theorem

If $\mathbf{L}$ *is any algebra in a variety* $\mathcal{V}$ *of e-cyclic semilinear residuated lattices, then* $\mathcal{O}(\mathbf{L})$ *is an orthocomplete dense extension of* $\mathbf{L}$ *that belongs to* $\mathcal{V}$*.*

Task 4.3: We investigate the existence and uniqueness of minimal extensions of a GMV algebra, which are laterally complete, projectable, strongly projectable, or orthocomplete. We refer to these extensions as *hulls*. In this direction we prove the following results:

Theorem

Any algebra $\mathbf{L}$ *in a variety* $\mathcal{V}$ *of semilinear GMV algebras has a unique, up to isomorphism, laterally complete hull that belongs to* $\mathcal{V}$*.*

Theorem

Any algebra $\mathbf{L}$ *in a variety* $\mathcal{V}$ *of semilinear GMV algebras has a unique, up to isomorphism, projectable hull, strongly projectable hull, and orthocomplete hull that belongs to* $\mathcal{V}$*.*

Moreover, we generalise in the context of GMV algebras a well-known result for ℓ-groups, namely lateral completeness and projectability imply strong projectability, and therefore we obtain:

Theorem

If a GMV algebra is laterally complete and projectable, then it is orthocomplete.

WP 7

Canonical Formulas

Involved groups

- University of Amsterdam (The Netherlands)
- University of Salerno (Italy)
- Vienna University of Technology (Austria)
- University of Milan (Italy)
- University of Bern (Switzerland)
- University of Denver Colorado (United States)
- New Mexico University (United States)
- Latrobe University (Australia)
- Kyoto University (Japan)
- The Australian National University (Australia)
- Nanyang Technological University (Singapore)

Overall goal of the WP and its tasks

The global objective of this Work Package is to advance the state of the art in Canonical formulas for substructural logics.
The WP has the following tasks:

1 Extend the method of canonical formulas to substructural logics.

2.1 Study of stable modal logics.

2.2 Algebraic criteria for intermediate logics to admit analytic hypersequent calculi.

3 The bounded proof property though one-step algebras.

Task 1:

N. Bezhanishvili, N. Galatos, L. Spada.
Canonical formulas for k-potent commutative, integral residuated lattices. Algebra Universalis, 77(3), pp. 321-343, 2017.

We made first steps towards extending the methods of canonical formulas to substructural logics. In particular, we extended it to k-potent commutative integral residuated lattices. We also axiomatised (using these formulas) a large class of logics with the finite model property.

Task 2.1:


G. Bezhanishvili, N. Bezhanishvili, J. Ilin.
Stable modal logics. Provisionally accepted at the Review of Symbolic Logic.


G. Bezhanishvili, N. Bezhanishvili, J. Ilin.
Subframization and stabilization for superintuitionistic logics. Provisionally accepted at Journal of Logic and Computation.

We conducted a through study of stable modal logics. These are logics axiomatisable by canonical formulas and rules of a particular shape. All these logics have the finite and bounded proof property. Many new characterizations of these logics have been obtained. We also gave new characterization of subframe and stable intermediate logics (these are logics axiomatisable by particular kinds of canonical formulas) using the translation into intuitionistic modal logic.

Task 2.2:

F. M. Lauridsen

Intermediate logics admitting a structural hypersequent calculus. Accepted for publication in Studia Logica.

We provide an algebraic criterion for an intermediate logic to admit an analytic hypersequent calculus in terms of closure conditions of the corresponding variety of Heyting algebras. This leads to the following characterisation: an intermediate logic admits a cut-free structural hypersequent calculus if and only if it is axiomatised by canonical formulas of a particular shape, again connecting the method of canonical formulas with the proof theoretic analysis of intermediate (substructural) logics as anticipated in this work package.

Task 3:

N. Bezhanishvili, S. Ghilardi and F. M. Lauridsen, *One-step Heyting algebras and hypersequent calculi with the bounded proof property*. To appear in Journal of Logic and Computation.

We have made substantial progress towards investigating proof-theoretic properties of hypersequent calculi for intermediate logics using algebraic methods. We considered a new weakly analytic subformula property (the bounded proof property) of such calculi. We introduced one-step Heyting algebras and established a semantic criterion characterising calculi for intermediate logics with the bounded proof property and the finite model property in terms of one-step Heyting algebras. We showed how this semantic criterion can be applied to a number of calculi for well-known intermediate logics such as LC, KC and BD_2.

WP 8

Dualities

Involved groups

- University of Milan (Italy)
- University of Salerno (Italy)
- University of Barcelona (Spain)
- Vienna University of Technology (Austria)
- University of the Witswatersrand (South Africa)
- CONICET (Argentina)
- New Mexico State University (United States)
- University of Denver Colorado (United States)
- Vanderbilt University (United States)
- La Trobe University (Australia)

Overall goal of the WP and its tasks

The global objective of this Work Package is to advance the state of the art in Duality theory for substructural logics
The WP has the following tasks:

1 Duality for residuated lattices.

3 Join completions.

Task 1:

P. Jipsen, L. Spada.

A topological duality for resiudated lattices. In progress.

In this paper we describe combinatorial structures, called residuated contexts that are dual to the category of residuated lattices. This duality puts in a unified perspective several important results in the theory of residuated lattices that make use of combinatorial constructions in order to build the desired residuated lattice.

Task 3:

J. Gil-Férez, L. Spada, C. Tsinakis, and H. Zhou. *Join-Completions of Ordered Algebras*. Submitted.

We present a systematic study of join-extensions and join-completions of ordered algebras, which naturally leads to a refined and simplified treatment of fundamental results and constructions in the theory of ordered structures ranging from properties of the Dedekind-MacNeille completion to the proof of the finite embeddability property for a number of varieties of ordered algebras.

WP 9

Residuated lattices with operators

Involved groups

- CONICET (Argentina)
- IIIA-CSIC (Spain)
- University of Salerno (Italy)
- Autonomous University of Barcelona (Spain)
- University of Olomouc (Czech Republic)
- Institute of Computer Science (Czech Republic)
- University of Denver Colorado (United States)
- Latrobe University (Australia)
- The Australian National University (Australia)
- State University of Campinas (Brazil)

Overall goal of the WP and its tasks

The global objective of this Work Package is to study the algebraic semantics of modal extensions of substructural logics
The WP has the following tasks:

1. Modal extensions of monoidal t-norm fuzzy logics.
2. Expansion of residuated lattices with additional operations.
3. Minimal fuzzy modal logic over Lukasiewicz logic ML and its corresponding multi-modal logic mML.

Task 1:

A. Vidal, F. Bou, F. Esteva, and L. Godo.
On strong standard completeness in some MTLΔ expansions.
Soft Computing, 21(1): 125-147 (2017)

A. Vidal, F. Esteva, and L. Godo.
On modal extensions of product fuzzy logic. Journal of Logic and Computation, 27(1): 299-336 (2017).

We have considered modal expansions of MTL with additional rational truth-constants and the Delta operator. The intended (standard) semantics is given by Kripke models with crisp accessibility relations and taking the unit real interval [0, 1] as set of truth-values.

As a preliminary step we introduced a strongly complete axiomatic system for the non-modal MTL with a unique infinitary inference rule, called density rule, solving several open problems in the literature.

A second contribution has been to propose an alternative axiomatic system (with a countable number of "well behaved" infinitary rules) that coincides with the previous one for a large family of left-continuous t-norms, including the Product t-norm and all ordinal sums of Product and Ł ukasiewicz components. The main result provides, for any such a t-norm, a complete axiomatic modal extensions with respect to the local and global Kripke semantics. We also studied extensions and applications of these logics.

Task 2:

R. C. Ertola-Biraben, F. Esteva, L. Godo.
Expanding FLew with a Boolean connective. Soft Comput. 21(1): 97-111 (2017)

We studied the expansion of the class of residuated lattices with an additional operation that gives the greatest complemented element below a given argument, and proved that the corresponding expanded class is an equational class. We also defined the corresponding logic as an expansion of FLew with a unary connective and proved that it is conservative and has the Finite Model Property.

A similar study has been carried out with regular rather than complemented elements. In such a case, residuated lattices (and weaker structures) have been expanded with an additional operator providing the greatest regular below (Work in progress).

Task 3:

F. Esteva, L. Godo, R. O. Rodriguez.
On the relation between modal and multi-modal logics over Łukasiewicz logic. Proc. of FUZZ-IEEE 2017: 1-6,

We studied a technical question related to the minimal fuzzy modal logic over Lukasiewicz logic *ML* and its corresponding multi-modal logic mML (with a modality for each rational value in [0, 1]). We have proved that the (standard) tautologies of the modal logic *ML* (resp. *mML*) are in fact the common tautologies of all the logics the n-valued modal logic *MLn* (resp. all the *n*-valued logics *mMLn*) when letting n vary over the naturals. This fact opens the door to show an alternative proof of the finite model property for these logics and hence their decidability.