Topological dynamics of stable groups

Ludomir Newelski

Instytut Matematyczny
Uniwersytet Wrocławski

June 2013

Set-up

T is a stable theory in language L

$\mathfrak{C}$ is a monster model of T

$M \prec \mathfrak{C}$

G is a group definable in M

$S_G(M) = \{tp(a/M) : a \in G^{\mathfrak{C}}\}$

Definition

Let $p, q \in S_G(M)$

$p * q = tp(a \cdot b/M)$, where $a \models p$, $b \models q$ and $a \perp_M b$

- $(S_G(M), *)$ is a semi-group
- $Gen := \{p \in S_G(M) : p \text{ is generic}\}$ is a maximal subgroup of $S_G(M)$,
  a minimal left ideal in $S_G(M)$.
- $Gen \cong G^{\mathfrak{C}}/(G^0)^{\mathfrak{C}}$

Set-up

T is a stable theory in language L
$\mathfrak{C}$ is a monster model of T
$M \prec \mathfrak{C}$
G is a group definable in M
$S_G(M) = \{tp(a/M) : a \in G^{\mathfrak{C}}\}$

Definition

Let $p, q \in S_G(M)$
$p * q = tp(a \cdot b/M)$, where $a \models p$, $b \models q$ and $a \downarrow_M b$

- $(S_G(M), *)$ is a semi-group
- $Gen := \{p \in S_G(M) : p \text{ is generic}\}$ is a maximal subgroup of $S_G(M)$,
  a minimal left ideal in $S_G(M)$.
- $Gen \cong G^{\mathfrak{C}}/(G^0)^{\mathfrak{C}}$

Set-up

T is a stable theory in language L
$\mathfrak{C}$ is a monster model of T
$M \prec \mathfrak{C}$
G is a group definable in M
$S_G(M) = \{tp(a/M) : a \in G^{\mathfrak{C}}\}$

Definition

Let $p, q \in S_G(M)$
$p * q = tp(a \cdot b/M)$, where $a \models p$, $b \models q$ and $a \mathop{\smile\!\!\!\!\!\!\,|}_M b$

- $(S_G(M), *)$ is a semi-group
- $Gen := \{p \in S_G(M) : p \text{ is generic}\}$ is a maximal subgroup of $S_G(M)$,
  a minimal left ideal in $S_G(M)$.
- $Gen \cong G^{\mathfrak{C}}/(G^0)^{\mathfrak{C}}$

Set-up

T is a stable theory in language L
$\mathfrak{C}$ is a monster model of T
$M \prec \mathfrak{C}$
G is a group definable in M
$S_G(M) = \{tp(a/M) : a \in G^{\mathfrak{C}}\}$

Definition

Let $p, q \in S_G(M)$
$p * q = tp(a \cdot b/M)$, where $a \models p$, $b \models q$ and $a \perp_M b$

- $(S_G(M), *)$ is a semi-group
- $Gen := \{p \in S_G(M) : p \text{ is generic}\}$ is a maximal subgroup of $S_G(M)$,
  a minimal left ideal in $S_G(M)$.
- $Gen \cong G^{\mathfrak{C}}/(G^0)^{\mathfrak{C}}$

Set-up

T is a stable theory in language L
$\mathfrak{C}$ is a monster model of T
$M \prec \mathfrak{C}$
G is a group definable in M
$S_G(M) = \{tp(a/M) : a \in G^{\mathfrak{C}}\}$

Definition

Let $p, q \in S_G(M)$
$p * q = tp(a \cdot b/M)$, where $a \models p$, $b \models q$ and $a \perp_M b$

- $(S_G(M), *)$ is a semi-group
- $Gen := \{p \in S_G(M) : p \text{ is generic}\}$ is a maximal subgroup of $S_G(M)$,
  a minimal left ideal in $S_G(M)$.
- $Gen \cong G^{\mathfrak{C}}/(G^0)^{\mathfrak{C}}$

Set-up

T is a stable theory in language L
$\mathfrak{C}$ is a monster model of T
$M \prec \mathfrak{C}$
G is a group definable in M
$S_G(M) = \{tp(a/M) : a \in G^{\mathfrak{C}}\}$

Definition

Let $p, q \in S_G(M)$
$p * q = tp(a \cdot b/M)$, where $a \models p$, $b \models q$ and $a \downarrow_M b$

- $(S_G(M), *)$ is a semi-group
- $Gen := \{p \in S_G(M) : p \text{ is generic}\}$ is a maximal subgroup of $S_G(M)$,
  a minimal left ideal in $S_G(M)$.
- $Gen \cong G^{\mathfrak{C}}/(G^0)^{\mathfrak{C}}$

Set-up

T is a stable theory in language L
$\mathfrak{C}$ is a monster model of T
$M \prec \mathfrak{C}$
G is a group definable in M
$S_G(M) = \{tp(a/M) : a \in G^{\mathfrak{C}}\}$

Definition

Let $p, q \in S_G(M)$
$p * q = tp(a \cdot b/M)$, where $a \models p$, $b \models q$ and $a \perp_M b$

- $(S_G(M), *)$ is a semi-group
- $Gen := \{p \in S_G(M) : p \text{ is generic}\}$ is a maximal subgroup of $S_G(M)$,
  a minimal left ideal in $S_G(M)$.
- $Gen \cong G^{\mathfrak{C}}/(G^0)^{\mathfrak{C}}$

Set-up

T is a stable theory in language L
$\mathfrak{C}$ is a monster model of T
$M \prec \mathfrak{C}$
G is a group definable in M
$S_G(M) = \{tp(a/M) : a \in G^{\mathfrak{C}}\}$

Definition

Let $p, q \in S_G(M)$
$p * q = tp(a \cdot b/M)$, where $a \models p$, $b \models q$ and $a \perp_M b$

- $(S_G(M), *)$ is a semi-group
- $Gen := \{p \in S_G(M) : p \text{ is generic}\}$ is a maximal subgroup of $S_G(M)$,
  a minimal left ideal in $S_G(M)$.
- $Gen \cong G^{\mathfrak{C}}/(G^0)^{\mathfrak{C}}$

Set-up

T is a stable theory in language L
$\mathfrak{C}$ is a monster model of T
$M \prec \mathfrak{C}$
G is a group definable in M
$S_G(M) = \{tp(a/M) : a \in G^{\mathfrak{C}}\}$

Definition

Let $p, q \in S_G(M)$
$p * q = tp(a \cdot b/M)$, where $a \models p$, $b \models q$ and $a \perp_M b$

- $(S_G(M), *)$ is a semi-group
- $Gen := \{p \in S_G(M) : p \text{ is generic}\}$ is a maximal subgroup of $S_G(M)$,
  a minimal left ideal in $S_G(M)$.
- $Gen \cong G^{\mathfrak{C}}/(G^0)^{\mathfrak{C}}$

A result

Theorem 1

$(S_G(M), *)$ is an inverse limit of a definable system of type-definable semigroups (in M^{eq}).

The proof uses:

- the definability lemma in local stability theory (the full version, Pillay)
- topological dynamics, particularly the functional representation of G-types.

A result

Theorem 1

$(S_G(M), *)$ is an inverse limit of a definable system of type-definable semigroups (in M^{eq}).

The proof uses:

- the definability lemma in local stability theory (the full version, Pillay)
- topological dynamics, particularly the functional representation of G-types.

A result

Theorem 1

$(S_G(M), *)$ is an inverse limit of a definable system of type-definable semigroups (in M^{eq}).

The proof uses:

- the definability lemma in local stability theory (the full version, Pillay)
- topological dynamics, particularly the functional representation of G-types.

A result

Theorem 1

$(S_G(M), *)$ is an inverse limit of a definable system of type-definable semigroups (in M^{eq}).

The proof uses:

- the definability lemma in local stability theory (the full version, Pillay)
- topological dynamics, particularly the functional representation of G-types.

Definition

(1) X is a G-flow if

- X is a compact topological space
- G acts on X by homeomorphisms

(2) X is point-transitive if there is a dense G-orbit $\subseteq X$.
(3) $Y \subseteq X$ is a G-subflow of X if Y is closed and G-closed.

Example

Let X be a G-flow and $p \in X$. Then $cl(Gp)$ is a subflow of X generated by p.

Definition

(1) X is a G-flow if

- X is a compact topological space
- G acts on X by homeomorphisms

(2) X is point-transitive if there is a dense G-orbit $\subseteq X$.

(3) $Y \subseteq X$ is a G-subflow of X if Y is closed and G-closed.

Example

Let X be a G-flow and $p \in X$. Then $cl(Gp)$ is a subflow of X generated by p.

Definition

(1) X is a G-flow if

- X is a compact topological space
- G acts on X by homeomorphisms

(2) X is point-transitive if there is a dense G-orbit $\subseteq X$.

(3) $Y \subseteq X$ is a G-subflow of X if Y is closed and G-closed.

Example

Let X be a G-flow and $p \in X$. Then $cl(Gp)$ is a subflow of X generated by p.

Topological dynamics

Definition

(1) X is a G-flow if

- X is a compact topological space
- G acts on X by homeomorphisms

(2) X is point-transitive if there is a dense G-orbit $\subseteq X$.
(3) $Y \subseteq X$ is a G-subflow of X if Y is closed and G-closed.

Example

Let X be a G-flow and $p \in X$. Then $cl(Gp)$ is a subflow of X generated by p.

Definition

(1) X is a G-flow if

- X is a compact topological space
- G acts on X by homeomorphisms

(2) X is point-transitive if there is a dense G-orbit $\subseteq X$.
(3) $Y \subseteq X$ is a G-subflow of X if Y is closed and G-closed.

Example

Let X be a G-flow and $p \in X$. Then $cl(Gp)$ is a subflow of X generated by p.

Definition continued

Assume X is a G-flow and $p \in X$.

(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.

(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.

(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.

(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.

$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$

$Gen(X) = \{p \in X : p \text{ is generic}\}$

$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Topological dynamics

Definition continued

Assume X is a G-flow and $p \in X$.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X) U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Definition continued

Assume X is a G-flow and $p \in X$.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Topological dynamics

Definition continued

Assume X is a G-flow and $p \in X$.
(4) p is almost periodic if $cl(Gp)$ is a minimal subflow of X.
(5) $U \subseteq X$ is generic if $(\exists A \subseteq_{fin} G)AU = X$.
(6) $U \subseteq X$ is weakly generic if $(\exists V \subseteq X)U \cup V$ is generic and V is non-generic.
(7) p is [weakly] generic if every open $U \ni p$ is.

Assume X is a G-flow.
$WGen(X) = \{p \in X : p \text{ is weakly generic}\}$
$Gen(X) = \{p \in X : p \text{ is generic}\}$
$APer(X) = \{p \in X : p \text{ is almost periodic}\}$

Fact

- $APer(X) = \bigcup\{\text{minimal subflows of } X\}$
- $APer(X) \neq \emptyset$
- $WGen(X) = cl(APer(X))$
- If $Gen(X) \neq \emptyset$, then $Gen(X) = WGen(X) = APer(X)$
- $Gen(X) \neq \emptyset$ iff there is just one minimal subflow of X.

Fact

- $APer(X) = \bigcup\{\text{minimal subflows of } X\}$
- $APer(X) \neq \emptyset$
- $WGen(X) = cl(APer(X))$
- If $Gen(X) \neq \emptyset$, then $Gen(X) = WGen(X) = APer(X)$
- $Gen(X) \neq \emptyset$ iff there is just one minimal subflow of X.

Topological dynamics

Fact

- $APer(X) = \bigcup\{\text{minimal subflows of } X\}$
- $APer(X) \neq \emptyset$
- $WGen(X) = cl(APer(X))$
- If $Gen(X) \neq \emptyset$, then $Gen(X) = WGen(X) = APer(X)$
- $Gen(X) \neq \emptyset$ iff there is just one minimal subflow of X.

Fact

- $APer(X) = \bigcup\{\text{minimal subflows of } X\}$
- $APer(X) \neq \emptyset$
- $WGen(X) = cl(APer(X))$
- If $Gen(X) \neq \emptyset$, then $Gen(X) = WGen(X) = APer(X)$
- $Gen(X) \neq \emptyset$ iff there is just one minimal subflow of X.

Topological dynamics

Fact

- $APer(X) = \bigcup\{\text{minimal subflows of } X\}$
- $APer(X) \neq \emptyset$
- $WGen(X) = cl(APer(X))$
- If $Gen(X) \neq \emptyset$, then $Gen(X) = WGen(X) = APer(X)$
- $Gen(X) \neq \emptyset$ iff there is just one minimal subflow of X.

Model theory

Let $Def_G(M) = \{$definable subsets of $G\}$.

- $Def_G(M)$ is an algebra od sets, closed under left translation in G
- $S_G(M) = S(Def_G(M))$
- G acts on $S_G(M)$ by left translation:

$$g \cdot p = \{\varphi(g^{-1} \cdot x) : \varphi(x) \in p\}$$

- $S_G(M)$ is a point-transitive G-flow, the set $\{tp(g/M : g \in G\}$ is a dense orbit.
- $p \in S_G(M)$ is a generic type iff p is a generic point in the G-flow $S_G(M)$.

Let $Def_G(M) = \{\text{definable subsets of } G\}$.

- $Def_G(M)$ is an algebra od sets, closed under left translation in G
- $S_G(M) = S(Def_G(M))$
- G acts on $S_G(M)$ by left translation:

$$g \cdot p = \{\varphi(g^{-1} \cdot x) : \varphi(x) \in p\}$$

- $S_G(M)$ is a point-transitive G-flow, the set $\{tp(g/M : g \in G\}$ is a dense orbit.
- $p \in S_G(M)$ is a generic type iff p is a generic point in the G-flow $S_G(M)$.

Model theory

Let $Def_G(M) = \{\text{definable subsets of } G\}$.

- $Def_G(M)$ is an algebra od sets, closed under left translation in G
- $S_G(M) = S(Def_G(M))$
- G acts on $S_G(M)$ by left translation:

$$g \cdot p = \{\varphi(g^{-1} \cdot x) : \varphi(x) \in p\}$$

- $S_G(M)$ is a point-transitive G-flow,
  the set $\{tp(g/M : g \in G\}$ is a dense orbit.
- $p \in S_G(M)$ is a generic type iff p is a generic point in the G-flow $S_G(M)$.

Model theory

Let $Def_G(M) = \{\text{definable subsets of } G\}$.

- $Def_G(M)$ is an algebra od sets, closed under left translation in G
- $S_G(M) = S(Def_G(M))$
- G acts on $S_G(M)$ by left translation:

$$g \cdot p = \{\varphi(g^{-1} \cdot x) : \varphi(x) \in p\}$$

- $S_G(M)$ is a point-transitive G-flow, the set $\{tp(g/M : g \in G\}$ is a dense orbit.
- $p \in S_G(M)$ is a generic type iff p is a generic point in the G-flow $S_G(M)$.

Model theory

Let $Def_G(M) = \{\text{definable subsets of } G\}$.

- $Def_G(M)$ is an algebra od sets, closed under left translation in G
- $S_G(M) = S(Def_G(M))$
- G acts on $S_G(M)$ by left translation:

$$g \cdot p = \{\varphi(g^{-1} \cdot x) : \varphi(x) \in p\}$$

- $S_G(M)$ is a point-transitive G-flow, the set $\{tp(g/M : g \in G\}$ is a dense orbit.
- $p \in S_G(M)$ is a generic type iff p is a generic point in the G-flow $S_G(M)$.

Model theory

Let $Def_G(M) = \{$definable subsets of $G\}$.

- $Def_G(M)$ is an algebra od sets, closed under left translation in G
- $S_G(M) = S(Def_G(M))$
- G acts on $S_G(M)$ by left translation:

$$g \cdot p = \{\varphi(g^{-1} \cdot x) : \varphi(x) \in p\}$$

- $S_G(M)$ is a point-transitive G-flow,
  the set $\{tp(g/M : g \in G\}$ is a dense orbit.
- $p \in S_G(M)$ is a generic type iff p is a generic point in the G-flow $S_G(M)$.

Model theory

Let $Def_G(M) = \{\text{definable subsets of } G\}$.

- $Def_G(M)$ is an algebra od sets, closed under left translation in G
- $S_G(M) = S(Def_G(M))$
- G acts on $S_G(M)$ by left translation:

$$g \cdot p = \{\varphi(g^{-1} \cdot x) : \varphi(x) \in p\}$$

- $S_G(M)$ is a point-transitive G-flow,
  the set $\{tp(g/M : g \in G\}$ is a dense orbit.
- $p \in S_G(M)$ is a generic type iff p is a generic point in the G-flow $S_G(M)$.

Model theory

Let $Def_G(M) = \{\text{definable subsets of } G\}$.

- $Def_G(M)$ is an algebra od sets, closed under left translation in G
- $S_G(M) = S(Def_G(M))$
- G acts on $S_G(M)$ by left translation:

$$g \cdot p = \{\varphi(g^{-1} \cdot x) : \varphi(x) \in p\}$$

- $S_G(M)$ is a point-transitive G-flow,
  the set $\{tp(g/M : g \in G\}$ is a dense orbit.
- $p \in S_G(M)$ is a generic type iff p is a generic point in the G-flow $S_G(M)$.

Topological dynamics

Let X be a point-transitive G-flow.

$G \ni g \rightsquigarrow \pi_g : X \overset{\approx}{\to} X,\ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r.t. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a point-transitive G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Topological dynamics

Let X be a point-transitive G-flow.
$G \ni g \rightsquigarrow \pi_g : X \overset{\approx}{\to} X,\ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r.t. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a point-transitive G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Topological dynamics

Let X be a point-transitive G-flow.
$G \ni g \rightsquigarrow \pi_g : X \overset{\approx}{\to} X,\ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r.t. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a point-transitive G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Topological dynamics

Let X be a point-transitive G-flow.
$G \ni g \rightsquigarrow \pi_g : X \overset{\approx}{\to} X, \ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a point-transitive G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Topological dynamics

Let X be a point-transitive G-flow.
$G \ni g \ \rightsquigarrow \pi_g : X \overset{\approx}{\to} X,\ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r.t. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a point-transitive G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Topological dynamics

Let X be a point-transitive G-flow.
$G \ni g \rightsquigarrow \pi_g : X \overset{\approx}{\to} X,\ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a point-transitive G-flow:
  1. for $f \in E(X)$ and $g \in G$, $g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Topological dynamics

Let X be a point-transitive G-flow.
$G \ni g \rightsquigarrow \pi_g : X \overset{\approx}{\to} X,\ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a point-transitive G-flow:
  1. for $f \in E(X)$ and $g \in G,\ g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Topological dynamics

Let X be a point-transitive G-flow.
$G \ni g \rightsquigarrow \pi_g : X \stackrel{\approx}{\to} X,\ \pi_g(x) = g \cdot x,$

$$E(X) = cl(\{\pi_g : g \in G\}) \subseteq X^X$$

- cl is the topological closure w.r. to pointwise convergence topology in X^X
- $E(X)$ is the Ellis (enveloping) semigroup of X
- $E(X)$ is a point-transitive G-flow:
  1. for $f \in E(X)$ and $g \in G,\ g * f = \pi_g \circ f$
  2. $\{\pi_g : g \in G\}$ is a dense G-orbit.
- $\circ$ is continuous on $E(X)$, in the first coordinate.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Sometimes $X \cong E(X)$, as G-flows. For example, $E(S_G(M)) \cong S_G(M)$, as G-flows and semigroups.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Sometimes $X \cong E(X)$, as G-flows. For example,
$E(S_G(M)) \cong S_G(M)$, as G-flows and semigroups.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Sometimes $X \cong E(X)$, as G-flows. For example, $E(S_G(M)) \cong S_G(M)$, as G-flows and semigroups.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Sometimes $X \cong E(X)$, as G-flows. For example, $E(S_G(M)) \cong S_G(M)$, as G-flows and semigroups.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Sometimes $X \cong E(X)$, as G-flows. For example,
$E(S_G(M)) \cong S_G(M)$, as G-flows and semigroups.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Sometimes $X \cong E(X)$, as G-flows. For example, $E(S_G(M)) \cong S_G(M)$, as G-flows and semigroups.

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Sometimes $X \cong E(X)$, as G-flows. For example,
$E(S_G(M)) \cong S_G(M)$, as G-flows and semigroups.

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Sometimes $X \cong E(X)$, as G-flows. For example,
$E(S_G(M)) \cong S_G(M)$, as G-flows and semigroups.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Sometimes $X \cong E(X)$, as G-flows. For example,
$E(S_G(M)) \cong S_G(M)$, as G-flows and semigroups.

Ellis semigroup

Definition

1. $I \subseteq E(X)$ is an ideal if $I \neq \emptyset$ and $fI \subseteq I$ for every $f \in E(X)$.
2. $j \in E(X)$ is an idempotent if $j^2 = j$.

Properties of $E(X)$

- Minimal subflows of $E(X)$ = minimal ideals in $E(X)$.
- Let $I \subseteq E(X)$ be a minimal ideal and $j \in I$ be an idempotent. Then $jI \subseteq I$ is a group (with identity j), called an ideal subgroup of $E(X)$ and
  I is a union of its ideal subgroups.
- The ideal subgroups of $E(X)$ are isomorphic.
- $E(X)$ explains the structure of X.

Sometimes $X \cong E(X)$, as G-flows. For example,
$E(S_G(M)) \cong S_G(M)$, as G-flows and semigroups.

Functional representation

Let $\mathcal{A} \subseteq \mathcal{P}(G)$ be a G-algebra of sets
(i.e. closed under left translation in G).
Then $S(\mathcal{A})$ is a G-flow.
For $p \in S(\mathcal{A})$ we define $d_p : \mathcal{A} \to \mathcal{P}(G)$ by:

$$d_p(U) = \{g \in G : g^{-1}U \in p\}$$

Definition

$\mathcal{A}$ is d-closed if $\mathcal{A}$ is closed under d_p for every $p \in S(\mathcal{A})$.

Example

$\mathcal{A} = Def_G(M)$ is d-closed, because every $p \in S_G(M)$ is definable.

Functional representation

Let $\mathcal{A} \subseteq \mathcal{P}(G)$ be a G-algebra of sets
(i.e. closed under left translation in G).
Then $S(\mathcal{A})$ is a G-flow.
For $p \in S(\mathcal{A})$ we define $d_p : \mathcal{A} \to \mathcal{P}(G)$ by:

$$d_p(U) = \{g \in G : g^{-1}U \in p\}$$

Definition

$\mathcal{A}$ is *d-closed* if $\mathcal{A}$ is closed under d_p for every $p \in S(\mathcal{A})$.

Example

$\mathcal{A} = Def_G(M)$ is d-closed, because every $p \in S_G(M)$ is definable.

Functional representation

Let $\mathcal{A} \subseteq \mathcal{P}(G)$ be a G-algebra of sets (i.e. closed under left translation in G). Then $S(\mathcal{A})$ is a G-flow.

For $p \in S(\mathcal{A})$ we define $d_p : \mathcal{A} \to \mathcal{P}(G)$ by:

$$d_p(U) = \{g \in G : g^{-1}U \in p\}$$

Definition

$\mathcal{A}$ is *d-closed* if $\mathcal{A}$ is closed under d_p for every $p \in S(\mathcal{A})$.

Example

$\mathcal{A} = Def_G(M)$ is d-closed, because every $p \in S_G(M)$ is definable.

Functional representation

Let $\mathcal{A} \subseteq \mathcal{P}(G)$ be a G-algebra of sets
(i.e. closed under left translation in G).
Then $S(\mathcal{A})$ is a G-flow.
For $p \in S(\mathcal{A})$ we define $d_p : \mathcal{A} \to \mathcal{P}(G)$ by:

$$d_p(U) = \{g \in G : g^{-1}U \in p\}$$

Definition

$\mathcal{A}$ is *d-closed* if $\mathcal{A}$ is closed under d_p for every $p \in S(\mathcal{A})$.

Example

$\mathcal{A} = Def_G(M)$ is d-closed, because every $p \in S_G(M)$ is definable.

Functional representation

Let $\mathcal{A} \subseteq \mathcal{P}(G)$ be a G-algebra of sets (i.e. closed under left translation in G).
Then $S(\mathcal{A})$ is a G-flow.
For $p \in S(\mathcal{A})$ we define $d_p : \mathcal{A} \to \mathcal{P}(G)$ by:

$$d_p(U) = \{g \in G : g^{-1}U \in p\}$$

Definition

$\mathcal{A}$ is *d-closed* if $\mathcal{A}$ is closed under d_p for every $p \in S(\mathcal{A})$.

Example

$\mathcal{A} = Def_G(M)$ is d-closed, because every $p \in S_G(M)$ is definable.

Functional representation

Let $\mathcal{A} \subseteq \mathcal{P}(G)$ be a G-algebra of sets
(i.e. closed under left translation in G).
Then $S(\mathcal{A})$ is a G-flow.
For $p \in S(\mathcal{A})$ we define $d_p : \mathcal{A} \to \mathcal{P}(G)$ by:

$$d_p(U) = \{g \in G : g^{-1}U \in p\}$$

Definition

$\mathcal{A}$ is *d-closed* if $\mathcal{A}$ is closed under d_p for every $p \in S(\mathcal{A})$.

Example

$\mathcal{A} = Def_G(M)$ is d-closed, because every $p \in S_G(M)$ is definable.

Functional representation

Let $\mathcal{A} \subseteq \mathcal{P}(G)$ be a G-algebra of sets
(i.e. closed under left translation in G).
Then $S(\mathcal{A})$ is a G-flow.
For $p \in S(\mathcal{A})$ we define $d_p : \mathcal{A} \to \mathcal{P}(G)$ by:

$$d_p(U) = \{g \in G : g^{-1}U \in p\}$$

Definition

$\mathcal{A}$ is *d-closed* if $\mathcal{A}$ is closed under d_p for every $p \in S(\mathcal{A})$.

Example

$\mathcal{A} = Def_G(M)$ is d-closed, because every $p \in S_G(M)$ is definable.

Functional representation

Assume $\mathcal{A}$ is d-closed.

- For $p \in S(\mathcal{A})$, $d_p \in End(\mathcal{A}) := \{G\text{-endomorphisms of } \mathcal{A}\}$.
- Let $d : S(\mathcal{A}) \to End(\mathcal{A})$ map p to d_p. Then d is a bijection.
- d induces $*$ on $S(\mathcal{A})$ so that

$$d : (S(\mathcal{A}), *) \stackrel{\cong}{\to} (End(\mathcal{A}), \circ)$$

Theorem 2

$$(E(S(\mathcal{A})), \circ) \cong^1 (S(\mathcal{A}), *) \cong^2 (End(\mathcal{A}), \circ)$$

Proof

1. For $p \in S(\mathcal{A})$ let $l_p(q) = p * q$.
Then $l_p \in E(S(\mathcal{A}))$ and $p \mapsto l_p$ gives $\cong^1$.
2. This is d.

Functional representation

Assume $\mathcal{A}$ is d-closed.

- For $p \in S(\mathcal{A})$, $d_p \in End(\mathcal{A}) := \{G\text{-endomorphisms of } \mathcal{A}\}$.
- Let $d : S(\mathcal{A}) \to End(\mathcal{A})$ map p to d_p. Then d is a bijection.
- d induces $*$ on $S(\mathcal{A})$ so that

$$d : (S(\mathcal{A}), *) \overset{\cong}{\to} (End(\mathcal{A}), \circ)$$

Theorem 2

$$(E(S(\mathcal{A})), \circ) \cong^1 (S(\mathcal{A}), *) \cong^2 (End(\mathcal{A}), \circ)$$

Proof

1. For $p \in S(\mathcal{A})$ let $l_p(q) = p * q$.
Then $l_p \in E(S(\mathcal{A}))$ and $p \mapsto l_p$ gives $\cong^1$.
2. This is d.

Functional representation

Assume $\mathcal{A}$ is d-closed.

- For $p \in S(\mathcal{A})$, $d_p \in End(\mathcal{A}) := \{G\text{-endomorphisms of } \mathcal{A}\}$.
- Let $d : S(\mathcal{A}) \to End(\mathcal{A})$ map p to d_p. Then d is a bijection.
- d induces $*$ on $S(\mathcal{A})$ so that

$$d : (S(\mathcal{A}), *) \overset{\cong}{\to} (End(\mathcal{A}), \circ)$$

Theorem 2

$$(E(S(\mathcal{A})), \circ) \cong^1 (S(\mathcal{A}), *) \cong^2 (End(\mathcal{A}), \circ)$$

Proof

1. For $p \in S(\mathcal{A})$ let $l_p(q) = p * q$.
Then $l_p \in E(S(\mathcal{A}))$ and $p \mapsto l_p$ gives $\cong^1$.
2. This is d.

Functional representation

Assume $\mathcal{A}$ is d-closed.

- For $p \in S(\mathcal{A})$, $d_p \in End(\mathcal{A}) := \{G\text{-endomorphisms of } \mathcal{A}\}$.
- Let $d : S(\mathcal{A}) \to End(\mathcal{A})$ map p to d_p. Then d is a bijection.
- d induces $*$ on $S(\mathcal{A})$ so that

$$d : (S(\mathcal{A}), *) \overset{\cong}{\to} (End(\mathcal{A}), \circ)$$

Theorem 2

$$(E(S(\mathcal{A})), \circ) \cong^1 (S(\mathcal{A}), *) \cong^2 (End(\mathcal{A}), \circ)$$

Proof

1. For $p \in S(\mathcal{A})$ let $l_p(q) = p * q$.
Then $l_p \in E(S(\mathcal{A}))$ and $p \mapsto l_p$ gives $\cong^1$.
2. This is d.

Functional representation

Assume $\mathcal{A}$ is d-closed.

- For $p \in S(\mathcal{A})$, $d_p \in End(\mathcal{A}) := \{G\text{-endomorphisms of } \mathcal{A}\}$.
- Let $d : S(\mathcal{A}) \to End(\mathcal{A})$ map p to d_p. Then d is a bijection.
- d induces $*$ on $S(\mathcal{A})$ so that

$$d : (S(\mathcal{A}), *) \overset{\cong}{\to} (End(\mathcal{A}), \circ)$$

Theorem 2

$$(E(S(\mathcal{A})), \circ) \cong^1 (S(\mathcal{A}), *) \cong^2 (End(\mathcal{A}), \circ)$$

Proof

1. For $p \in S(\mathcal{A})$ let $l_p(q) = p * q$.
Then $l_p \in E(S(\mathcal{A}))$ and $p \mapsto l_p$ gives $\cong^1$.
2. This is d.

Functional representation

Assume $\mathcal{A}$ is d-closed.

- For $p \in S(\mathcal{A})$, $d_p \in End(\mathcal{A}) := \{G\text{-endomorphisms of } \mathcal{A}\}$.
- Let $d : S(\mathcal{A}) \to End(\mathcal{A})$ map p to d_p. Then d is a bijection.
- d induces $*$ on $S(\mathcal{A})$ so that

$$d : (S(\mathcal{A}), *) \xrightarrow{\cong} (End(\mathcal{A}), \circ)$$

Theorem 2

$$(E(S(\mathcal{A})), \circ) \cong^1 (S(\mathcal{A}), *) \cong^2 (End(\mathcal{A}), \circ)$$

Proof

1. For $p \in S(\mathcal{A})$ let $l_p(q) = p * q$.
Then $l_p \in E(S(\mathcal{A}))$ and $p \mapsto l_p$ gives $\cong^1$.
2. This is d.

Functional representation

Assume $\mathcal{A}$ is d-closed.

- For $p \in S(\mathcal{A})$, $d_p \in End(\mathcal{A}) := \{G\text{-endomorphisms of } \mathcal{A}\}$.
- Let $d : S(\mathcal{A}) \to End(\mathcal{A})$ map p to d_p. Then d is a bijection.
- d induces $*$ on $S(\mathcal{A})$ so that

$$d : (S(\mathcal{A}), *) \overset{\cong}{\to} (End(\mathcal{A}), \circ)$$

Theorem 2

$$(E(S(\mathcal{A})), \circ) \cong^1 (S(\mathcal{A}), *) \cong^2 (End(\mathcal{A}), \circ)$$

Proof

1. For $p \in S(\mathcal{A})$ let $l_p(q) = p * q$.
Then $l_p \in E(S(\mathcal{A}))$ and $p \mapsto l_p$ gives $\cong^1$.
2. This is d.

Assume $\mathcal{A}$ is d-closed.

- For $p \in S(\mathcal{A})$, $d_p \in End(\mathcal{A}) := \{G\text{-endomorphisms of } \mathcal{A}\}$.
- Let $d : S(\mathcal{A}) \to End(\mathcal{A})$ map p to d_p. Then d is a bijection.
- d induces $*$ on $S(\mathcal{A})$ so that

$$d : (S(\mathcal{A}), *) \stackrel{\cong}{\to} (End(\mathcal{A}), \circ)$$

Theorem 2

$$(E(S(\mathcal{A})), \circ) \cong^1 (S(\mathcal{A}), *) \cong^2 (End(\mathcal{A}), \circ)$$

Proof

1. For $p \in S(\mathcal{A})$ let $l_p(q) = p * q$.
Then $l_p \in E(S(\mathcal{A}))$ and $p \mapsto l_p$ gives $\cong^1$.
2. This is d.

Functional representation

Example

If $\mathcal{A} = Def_G(M)$ then $\mathcal{A}$ is d-closed and $*$ on $S_G(M) = S(\mathcal{A})$ from Theorem 2 is just the free multiplication of G-types.

Functional representation

Example

If $\mathcal{A} = Def_G(M)$ then $\mathcal{A}$ is d-closed and $*$ on $S_G(M) = S(\mathcal{A})$ from Theorem 2 is just the free multiplication of G-types.

$(S_G(M), *)$ in the definable realm

Definition

1. $\Delta \subseteq L$ is invariant if the family of relatively Δ-definable subsets of G is closed under left and right translation in G.
2. Let $Inv = \{\Delta \subseteq_{fin} L : \Delta \text{ is invariant}\}$.

Fact

Inv is cofinal in $[L]^{<\omega}$.

Let $\Delta \in Inv$.

Notation

$$Def_{G,\Delta} = \{\text{relatively } \Delta\text{-definable subsets of } G\}$$

$$S_{G,\Delta} = S(Def_{G,\Delta}(M)),$$

the space of complete Δ-types of G over M.

$(S_G(M), *)$ in the definable realm

Definition

1. $\Delta \subseteq L$ is invariant if the family of relatively Δ-definable subsets of G is closed under left and right translation in G.
2. Let $Inv = \{\Delta \subseteq_{fin} L : \Delta \text{ is invariant}\}$.

Fact

Inv is cofinal in $[L]^{<\omega}$.

Let $\Delta \in Inv$.

Notation

$$Def_{G,\Delta} = \{\text{relatively } \Delta\text{-definable subsets of } G\}$$

$$S_{G,\Delta} = S(Def_{G,\Delta}(M)),$$

the space of complete Δ-types of G over M.

$(S_G(M), *)$ in the definable realm

Definition

1. $\Delta \subseteq L$ is invariant if the family of relatively Δ-definable subsets of G is closed under left and right translation in G.
2. Let $Inv = \{\Delta \subseteq_{fin} L : \Delta \text{ is invariant}\}$.

Fact

Inv is cofinal in $[L]^{<\omega}$.

Let $\Delta \in Inv$.

Notation

$$Def_{G,\Delta} = \{\text{relatively } \Delta\text{-definable subsets of } G\}$$

$$S_{G,\Delta} = S(Def_{G,\Delta}(M)),$$

the space of complete Δ-types of G over M.

$(S_G(M), *)$ in the definable realm

Definition

1. $\Delta \subseteq L$ is invariant if the family of relatively Δ-definable subsets of G is closed under left and right translation in G.
2. Let $Inv = \{\Delta \subseteq_{fin} L : \Delta \text{ is invariant}\}$.

Fact

Inv is cofinal in $[L]^{<\omega}$.

Let $\Delta \in Inv$.

Notation

$$Def_{G,\Delta} = \{\text{relatively } \Delta\text{-definable subsets of } G\}$$

$$S_{G,\Delta} = S(Def_{G,\Delta}(M)),$$

the space of complete Δ-types of G over M.

$(S_G(M), *)$ in the definable realm

Definition

1. $\Delta \subseteq L$ is invariant if the family of relatively Δ-definable subsets of G is closed under left and right translation in G.
2. Let $Inv = \{\Delta \subseteq_{fin} L : \Delta \text{ is invariant}\}$.

Fact

Inv is cofinal in $[L]^{<\omega}$.

Let $\Delta \in Inv$.

Notation

$$Def_{G,\Delta} = \{\text{relatively } \Delta\text{-definable subsets of } G\}$$

$$S_{G,\Delta} = S(Def_{G,\Delta}(M)),$$

the space of complete Δ-types of G over M.

$(S_G(M), *)$ in the definable realm

Definition

1. $\Delta \subseteq L$ is invariant if the family of relatively Δ-definable subsets of G is closed under left and right translation in G.
2. Let $Inv = \{\Delta \subseteq_{fin} L : \Delta \text{ is invariant}\}$.

Fact

Inv is cofinal in $[L]^{<\omega}$.

Let $\Delta \in Inv$.

Notation

$$Def_{G,\Delta} = \{\text{relatively } \Delta\text{-definable subsets of } G\}$$

$$S_{G,\Delta} = S(Def_{G,\Delta}(M)),$$

the space of complete Δ-types of G over M.

$(S_G(M), *)$ in the definable realm

Definition

1. $\Delta \subseteq L$ is invariant if the family of relatively Δ-definable subsets of G is closed under left and right translation in G.
2. Let $Inv = \{\Delta \subseteq_{fin} L : \Delta \text{ is invariant}\}$.

Fact

Inv is cofinal in $[L]^{<\omega}$.

Let $\Delta \in Inv$.

Notation

$$Def_{G,\Delta} = \{\text{relatively } \Delta\text{-definable subsets of } G\}$$

$$S_{G,\Delta} = S(Def_{G,\Delta}(M)),$$

the space of complete Δ-types of G over M.

$(S_G(M), *)$ in the definable realm

Definition

1. $\Delta \subseteq L$ is invariant if the family of relatively Δ-definable subsets of G is closed under left and right translation in G.
2. Let $Inv = \{\Delta \subseteq_{fin} L : \Delta \text{ is invariant}\}$.

Fact

Inv is cofinal in $[L]^{<\omega}$.

Let $\Delta \in Inv$.

Notation

$$Def_{G,\Delta} = \{\text{relatively } \Delta\text{-definable subsets of } G\}$$

$$S_{G,\Delta} = S(Def_{G,\Delta}(M)),$$

the space of complete Δ-types of G over M.

$(S_G(M), *)$ in the definable realm

Definition

1. $\Delta \subseteq L$ is invariant if the family of relatively Δ-definable subsets of G is closed under left and right translation in G.
2. Let $Inv = \{\Delta \subseteq_{fin} L : \Delta \text{ is invariant}\}$.

Fact

Inv is cofinal in $[L]^{<\omega}$.

Let $\Delta \in Inv$.

Notation

$$Def_{G,\Delta} = \{\text{relatively } \Delta\text{-definable subsets of } G\}$$

$$S_{G,\Delta} = S(Def_{G,\Delta}(M)),$$

the space of complete Δ-types of G over M.

$(S_G(M), *)$ in the definable realm

1. $Def_{G,\Delta}(M)$ is a d-closed G-algebra of sets.
   (this relies on the full definability lemma in local stability theory)
2. $(S_{G,\Delta}(M), *) \cong (E(S_{G,\Delta}(M)), \circ) \cong (End(Def_{G,\Delta}(M)), \circ)$
   (this is by Theorem 2)
3. $$Def_G(M) = \bigcup_{\Delta \in Inv} Def_{G,\Delta}(M)$$
4. $\langle S_{G,\Delta}(M), \Delta \in Inv \rangle$ is an inverse system of G- flows and semi-groups
   (the connecting functions are restrictions)
5. $S_G(M) = invlim_{\Delta \in Inv} S_{G,\Delta}(M)$
   (as G-flows and semigroups)

$(S_G(M), *)$ in the definable realm

1. $Def_{G,\Delta}(M)$ is a d-closed G-algebra of sets.
   (this relies on the full definability lemma in local stability theory)
2. $(S_{G,\Delta}(M), *) \cong (E(S_{G,\Delta}(M)), \circ) \cong (End(Def_{G,\Delta}(M)), \circ)$
   (this is by Theorem 2)
3. $$Def_G(M) = \bigcup_{\Delta \in Inv} Def_{G,\Delta}(M)$$
4. $\langle S_{G,\Delta}(M), \Delta \in Inv \rangle$ is an inverse system of G- flows and semi-groups
   (the connecting functions are restrictions)
5. $S_G(M) = invlim_{\Delta \in Inv} S_{G,\Delta}(M)$
   (as G-flows and semigroups)

$(S_G(M), *)$ in the definable realm

1. $Def_{G,\Delta}(M)$ is a d-closed G-algebra of sets.
   (this relies on the full definability lemma in local stability theory)
2. $(S_{G,\Delta}(M), *) \cong (E(S_{G,\Delta}(M)), \circ) \cong (End(Def_{G,\Delta}(M)), \circ)$
   (this is by Theorem 2)
3. $$Def_G(M) = \bigcup_{\Delta \in Inv} Def_{G,\Delta}(M)$$
4. $\langle S_{G,\Delta}(M), \Delta \in Inv \rangle$ is an inverse system of G- flows and semi-groups
   (the connecting functions are restrictions)
5. $S_G(M) = invlim_{\Delta \in Inv} S_{G,\Delta}(M)$
   (as G-flows and semigroups)

$(S_G(M), *)$ in the definable realm

1. $Def_{G,\Delta}(M)$ is a d-closed G-algebra of sets.
   (this relies on the full definability lemma in local stability theory)
2. $(S_{G,\Delta}(M), *) \cong (E(S_{G,\Delta}(M)), \circ) \cong (End(Def_{G,\Delta}(M)), \circ)$
   (this is by Theorem 2)
3. $$Def_G(M) = \bigcup_{\Delta \in Inv} Def_{G,\Delta}(M)$$
4. $\langle S_{G,\Delta}(M), \Delta \in Inv \rangle$ is an inverse system of G- flows and semi-groups
   (the connecting functions are restrictions)
5. $S_G(M) = invlim_{\Delta \in Inv} S_{G,\Delta}(M)$
   (as G-flows and semigroups)

$(S_G(M), *)$ in the definable realm

1. $Def_{G,\Delta}(M)$ is a d-closed G-algebra of sets.
   (this relies on the full definability lemma in local stability theory)
2. $(S_{G,\Delta}(M), *) \cong (E(S_{G,\Delta}(M)), \circ) \cong (End(Def_{G,\Delta}(M)), \circ)$
   (this is by Theorem 2)
3. $$Def_G(M) = \bigcup_{\Delta \in Inv} Def_{G,\Delta}(M)$$
4. $\langle S_{G,\Delta}(M), \Delta \in Inv \rangle$ is an inverse system of G- flows and semi-groups
   (the connecting functions are restrictions)
5. $S_G(M) = invlim_{\Delta \in Inv} S_{G,\Delta}(M)$
   (as G-flows and semigroups)

$(S_G(M), *)$ in the definable realm

1. $Def_{G,\Delta}(M)$ is a d-closed G-algebra of sets.
   (this relies on the full definability lemma in local stability theory)
2. $(S_{G,\Delta}(M), *) \cong (E(S_{G,\Delta}(M)), \circ) \cong (End(Def_{G,\Delta}(M)), \circ)$
   (this is by Theorem 2)
3. $$Def_G(M) = \bigcup_{\Delta \in Inv} Def_{G,\Delta}(M)$$
4. $\langle S_{G,\Delta}(M), \Delta \in Inv \rangle$ is an inverse system of G- flows and semi-groups
   (the connecting functions are restrictions)
5. $S_G(M) = invlim_{\Delta \in Inv} S_{G,\Delta}(M)$
   (as G-flows and semigroups)

$(S_G(M), *)$ in the definable realm

1. $Def_{G,\Delta}(M)$ is a d-closed G-algebra of sets.
   (this relies on the full definability lemma in local stability theory)
2. $(S_{G,\Delta}(M), *) \cong (E(S_{G,\Delta}(M)), \circ) \cong (End(Def_{G,\Delta}(M)), \circ)$
   (this is by Theorem 2)
3. $$Def_G(M) = \bigcup_{\Delta \in Inv} Def_{G,\Delta}(M)$$
4. $\langle S_{G,\Delta}(M), \Delta \in Inv \rangle$ is an inverse system of G- flows and semi-groups
   (the connecting functions are restrictions)
5. $S_G(M) = invlim_{\Delta \in Inv} S_{G,\Delta}(M)$
   (as G-flows and semigroups)

$(S_G(M), *)$ in the definable realm

1. $Def_{G,\Delta}(M)$ is a d-closed G-algebra of sets.
   (this relies on the full definability lemma in local stability theory)
2. $(S_{G,\Delta}(M), *) \cong (E(S_{G,\Delta}(M)), \circ) \cong (End(Def_{G,\Delta}(M)), \circ)$
   (this is by Theorem 2)
3. $$Def_G(M) = \bigcup_{\Delta \in Inv} Def_{G,\Delta}(M)$$
4. $\langle S_{G,\Delta}(M), \Delta \in Inv \rangle$ is an inverse system of G- flows and semi-groups
   (the connecting functions are restrictions)
5. $S_G(M) = invlim_{\Delta \in Inv} S_{G,\Delta}(M)$
   (as G-flows and semigroups)

Commuting diagram

$$\begin{array}{ccc}
(S_G(M), *) & \xrightarrow{r} & (S_{G,\Delta}(M), *) \\
d \downarrow \cong & & d \downarrow \cong \\
End(Def_G(M), \circ) & \xrightarrow{r} & End(Def_{G,\Delta}(M), \circ)
\end{array}$$

The horizontal arrows are restrictions.
All arrows are semigroup homomorphisms.

Commuting diagram

$$\begin{array}{ccc} (S_G(M), *) & \xrightarrow{\ r\ } & (S_{G,\Delta}(M), *) \\ d \downarrow \cong & & d \downarrow \cong \\ End(Def_G(M), \circ) & \xrightarrow{\ r\ } & End(Def_{G,\Delta}(M), \circ) \end{array}$$

The horizontal arrows are restrictions.

All arrows are semigroup homomorphisms.

Commuting diagram

$$\begin{array}{ccc} (S_G(M), *) & \xrightarrow{\quad r \quad} & (S_{G,\Delta}(M), *) \\ d \downarrow \cong & & d \downarrow \cong \\ End(Def_G(M), \circ) & \xrightarrow{r} & End(Def_{G,\Delta}(M), \circ) \end{array}$$

The horizontal arrows are restrictions.
All arrows are semigroup homomorphisms.

Proposition

$(S_{G,\Delta}(M), *)$ is a type-definable semigroup (in M^{eq}).

Proof.

- $S_{G,\Delta}(M)$ is a type-definable set in M^{eq}
  (identify $p \in S_{G,\Delta}(M)$ with the tuple of canonical φ-definitions of p, $\varphi \in \Delta$)
- $*$ is relatively definable on $S_{G,\Delta}(M)$.
  (Use $d : S_{G,\Delta}(M) \cong End(Def_{G,\Delta}(M))$, the full definability lemma and compactness.
  $Def_{G,\Delta}(M)$ is ind-definable in M^{eq}.)

$(S_G(M), *)$ in the definable realm

Proposition

$(S_{G,\Delta}(M), *)$ is a type-definable semigroup (in M^{eq}).

Proof.

- $S_{G,\Delta}(M)$ is a type-definable set in M^{eq} (identify $p \in S_{G,\Delta}(M)$ with the tuple of canonical φ-definitions of p, $\varphi \in \Delta$)
- $*$ is relatively definable on $S_{G,\Delta}(M)$. (Use $d : S_{G,\Delta}(M) \cong End(Def_{G,\Delta}(M))$, the full definability lemma and compactness. $Def_{G,\Delta}(M)$ is ind-definable in M^{eq}.)

$(S_G(M), *)$ in the definable realm

Proposition

$(S_{G,\Delta}(M), *)$ is a type-definable semigroup (in M^{eq}).

Proof.

- $S_{G,\Delta}(M)$ is a type-definable set in M^{eq}
  (identify $p \in S_{G,\Delta}(M)$ with the tuple of canonical φ-definitions of p, $\varphi \in \Delta$)
- $*$ is relatively definable on $S_{G,\Delta}(M)$.
  (Use $d : S_{G,\Delta}(M) \cong End(Def_{G,\Delta}(M))$, the full definability lemma and compactness.
  $Def_{G,\Delta}(M)$ is ind-definable in M^{eq}.)

$(S_G(M), *)$ in the definable realm

Proposition

$(S_{G,\Delta}(M), *)$ is a type-definable semigroup (in M^{eq}).

Proof.

- $S_{G,\Delta}(M)$ is a type-definable set in M^{eq}
  (identify $p \in S_{G,\Delta}(M)$ with the tuple of canonical φ-definitions of p, $\varphi \in \Delta$)
- $*$ is relatively definable on $S_{G,\Delta}(M)$.
  (Use $d : S_{G,\Delta}(M) \cong End(Def_{G,\Delta}(M))$, the full definability lemma and compactness.
  $Def_{G,\Delta}(M)$ is ind-definable in M^{eq}.)

$(S_G(M), *)$ in the definable realm

Proposition

$(S_{G,\Delta}(M), *)$ is a type-definable semigroup (in M^{eq}).

Proof.

- $S_{G,\Delta}(M)$ is a type-definable set in M^{eq}
(identify $p \in S_{G,\Delta}(M)$ with the tuple of canonical φ-definitions of p, $\varphi \in \Delta$)
- $*$ is relatively definable on $S_{G,\Delta}(M)$.
(Use $d : S_{G,\Delta}(M) \cong End(Def_{G,\Delta}(M))$, the full definability lemma and compactness.
$Def_{G,\Delta}(M)$ is ind-definable in M^{eq}.)

Proposition

$(S_{G,\Delta}(M), *)$ is a type-definable semigroup (in M^{eq}).

Proof.

- $S_{G,\Delta}(M)$ is a type-definable set in M^{eq} (identify $p \in S_{G,\Delta}(M)$ with the tuple of canonical φ-definitions of p, $\varphi \in \Delta$)
- $*$ is relatively definable on $S_{G,\Delta}(M)$. (Use $d : S_{G,\Delta}(M) \cong End(Def_{G,\Delta}(M))$, the full definability lemma and compactness. $Def_{G,\Delta}(M)$ is ind-definable in M^{eq}.)

$(S_G(M), *)$ in the definable realm

Proposition

$(S_{G,\Delta}(M), *)$ is a type-definable semigroup (in M^{eq}).

Proof.

- $S_{G,\Delta}(M)$ is a type-definable set in M^{eq}
  (identify $p \in S_{G,\Delta}(M)$ with the tuple of canonical φ-definitions of p, $\varphi \in \Delta$)
- $*$ is relatively definable on $S_{G,\Delta}(M)$.
  (Use $d : S_{G,\Delta}(M) \cong End(Def_{G,\Delta}(M))$, the full definability lemma and compactness.
  $Def_{G,\Delta}(M)$ is ind-definable in M^{eq}.)

Subgroups of $E(S(\mathcal{A})) \cong S(\mathcal{A}) \cong End(\mathcal{A})$

Assume $f \in End(\mathcal{A})$.

$$Ker(f) = \{U \in \mathcal{A} : f(U) = \emptyset\}$$

$$Im(f) = \{f(U) : U \in \mathcal{A}\}$$

- $Ker(f)$ is a G-ideal in $\mathcal{A}$.
- $Im(f)$ is a G-subalgebra of $\mathcal{A}$.

Crucial point

Assume $f, g \in End(\mathcal{A})$. Then

$$Ker(f \circ g) \supseteq Ker(g) \text{ and } Im(f \circ g) \subseteq Im(f)$$

Assume $f \in End(\mathcal{A})$.

$$Ker(f) = \{U \in \mathcal{A} : f(U) = \emptyset\}$$

$$Im(f) = \{f(U) : U \in \mathcal{A}\}$$

- $Ker(f)$ is a G-ideal in $\mathcal{A}$.
- $Im(f)$ is a G-subalgebra of $\mathcal{A}$.

Crucial point

Assume $f, g \in End(\mathcal{A})$. Then

$$Ker(f \circ g) \supseteq Ker(g) \text{ and } Im(f \circ g) \subseteq Im(f)$$

Subgroups of $E(S(\mathcal{A})) \cong S(\mathcal{A}) \cong End(\mathcal{A})$

Assume $f \in End(\mathcal{A})$.

$$Ker(f) = \{U \in \mathcal{A} : f(U) = \emptyset\}$$

$$Im(f) = \{f(U) : U \in \mathcal{A}\}$$

- $Ker(f)$ is a G-ideal in $\mathcal{A}$.
- $Im(f)$ is a G-subalgebra of $\mathcal{A}$.

Crucial point

Assume $f, g \in End(\mathcal{A})$. Then

$$Ker(f \circ g) \supseteq Ker(g) \text{ and } Im(f \circ g) \subseteq Im(f)$$

Subgroups of $E(S(\mathcal{A})) \cong S(\mathcal{A}) \cong End(\mathcal{A})$

Assume $f \in End(\mathcal{A})$.

$$Ker(f) = \{U \in \mathcal{A} : f(U) = \emptyset\}$$

$$Im(f) = \{f(U) : U \in \mathcal{A}\}$$

- $Ker(f)$ is a G-ideal in $\mathcal{A}$.
- $Im(f)$ is a G-subalgebra of $\mathcal{A}$.

Crucial point

Assume $f, g \in End(\mathcal{A})$. Then

$$Ker(f \circ g) \supseteq Ker(g) \text{ and } Im(f \circ g) \subseteq Im(f)$$

Assume $f \in End(\mathcal{A})$.

$$Ker(f) = \{U \in \mathcal{A} : f(U) = \emptyset\}$$

$$Im(f) = \{f(U) : U \in \mathcal{A}\}$$

- $Ker(f)$ is a G-ideal in $\mathcal{A}$.
- $Im(f)$ is a G-subalgebra of $\mathcal{A}$.

Crucial point

Assume $f, g \in End(\mathcal{A})$. Then

$$Ker(f \circ g) \supseteq Ker(g) \text{ and } Im(f \circ g) \subseteq Im(f)$$

Subgroups of $E(S(\mathcal{A})) \cong S(\mathcal{A}) \cong End(\mathcal{A})$

Assume $f \in End(\mathcal{A})$.

$$Ker(f) = \{U \in \mathcal{A} : f(U) = \emptyset\}$$

$$Im(f) = \{f(U) : U \in \mathcal{A}\}$$

- $Ker(f)$ is a G-ideal in $\mathcal{A}$.
- $Im(f)$ is a G-subalgebra of $\mathcal{A}$.

Crucial point

Assume $f, g \in End(\mathcal{A})$. Then

$$Ker(f \circ g) \supseteq Ker(g) \text{ and } Im(f \circ g) \subseteq Im(f)$$

Subgroups of $E(S(\mathcal{A})) \cong S(\mathcal{A}) \cong End(\mathcal{A})$

Assume $f \in End(\mathcal{A})$.

$$Ker(f) = \{U \in \mathcal{A} : f(U) = \emptyset\}$$

$$Im(f) = \{f(U) : U \in \mathcal{A}\}$$

- $Ker(f)$ is a G-ideal in $\mathcal{A}$.
- $Im(f)$ is a G-subalgebra of $\mathcal{A}$.

Crucial point

Assume $f, g \in End(\mathcal{A})$. Then

$$Ker(f \circ g) \supseteq Ker(g) \text{ and } Im(f \circ g) \subseteq Im(f)$$

Subgroups of $E(S(\mathcal{A})) \cong S(\mathcal{A}) \cong End(\mathcal{A})$

Assume $f \in End(\mathcal{A})$.

$$Ker(f) = \{U \in \mathcal{A} : f(U) = \emptyset\}$$

$$Im(f) = \{f(U) : U \in \mathcal{A}\}$$

- $Ker(f)$ is a G-ideal in $\mathcal{A}$.
- $Im(f)$ is a G-subalgebra of $\mathcal{A}$.

Crucial point

Assume $f, g \in End(\mathcal{A})$. Then

$$Ker(f \circ g) \supseteq Ker(g) \text{ and } Im(f \circ g) \subseteq Im(f)$$

Subgroups of $E(S(\mathcal{A})) \cong S(\mathcal{A}) \cong End(\mathcal{A})$

Proposition

Assume that S is a subgroup of $End(\mathcal{A})$. Then all $f \in S$ have a common kernel $K = K_S$ and common image $R = R_S$. Let $S_{K,R} = \{f \in End(\mathcal{A}) : Ker(f) = K,\ Im(f) = R \text{ and } f|_R \text{ permutes } R\}$. Then $S_{K,R}$ is a maximal subgroup of $End(\mathcal{A})$ containing S.

Let $\mathcal{I} = \{\text{ideal subgroups of } S(\mathcal{A})\}$
$\mathcal{K} = \{$ kernels of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$
$\mathcal{R} = \{$ images of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$

Fact

The mapping $\mathcal{I} \ni S \mapsto \langle K_S, R_S\rangle \in \mathcal{K} \times \mathcal{R}$ is a bijection $\mathcal{I} \to \mathcal{K} \times \mathcal{R}$.
The fibers of the surjective mapping $APer(S(\mathcal{A})) \ni p \mapsto Ker(d_p) \in \mathcal{K}$ are precisely the minimal subflows of $S(\mathcal{A})$.

Proposition

Assume that S is a subgroup of $End(\mathcal{A})$.Then all $f \in S$ have a common kernel $K = K_S$ and common image $R = R_S$.Let $S_{K,R} = \{f \in End(\mathcal{A}) : Ker(f) = K,\ Im(f) = R$ and $f|_R$ permutes $R\}$. Then $S_{K,R}$ is a maximal subgroup of $End(\mathcal{A})$ containing S.

Let $\mathcal{I} = \{$ideal subgroups of $S(\mathcal{A})\}$
$\mathcal{K} = \{$ kernels of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$
$\mathcal{R} = \{$ images of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$

Fact

The mapping $\mathcal{I} \ni S \mapsto \langle K_S, R_S \rangle \in \mathcal{K} \times \mathcal{R}$ is a bijection $\mathcal{I} \to \mathcal{K} \times \mathcal{R}$.
The fibers of the surjective mapping $APer(S(\mathcal{A})) \ni p \mapsto Ker(d_p) \in \mathcal{K}$ are precisely the minimal subflows of $S(\mathcal{A})$.

Proposition

Assume that S is a subgroup of $End(\mathcal{A})$.Then all $f \in S$ have a common kernel $K = K_S$ and common image $R = R_S$.Let $S_{K,R} = \{f \in End(\mathcal{A}) : Ker(f) = K,\ Im(f) = R$ and $f|_R$ permutes $R\}$. Then $S_{K,R}$ is a maximal subgroup of $End(\mathcal{A})$ containing S.

Let $\mathcal{I} = \{$ideal subgroups of $S(\mathcal{A})\}$
$\mathcal{K} = \{$ kernels of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$
$\mathcal{R} = \{$ images of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$

Fact

The mapping $\mathcal{I} \ni S \mapsto \langle K_S, R_S \rangle \in \mathcal{K} \times \mathcal{R}$ is a bijection $\mathcal{I} \to \mathcal{K} \times \mathcal{R}$.
The fibers of the surjective mapping $APer(S(\mathcal{A})) \ni p \mapsto Ker(d_p) \in \mathcal{K}$ are precisely the minimal subflows of $S(\mathcal{A})$.

Subgroups of $E(S(\mathcal{A})) \cong S(\mathcal{A}) \cong End(\mathcal{A})$

Proposition

Assume that S is a subgroup of $End(\mathcal{A})$.Then all $f \in S$ have a common kernel $K = K_S$ and common image $R = R_S$.Let $S_{K,R} = \{f \in End(\mathcal{A}) : Ker(f) = K,\ Im(f) = R \text{ and } f|_R \text{ permutes } R\}$. Then $S_{K,R}$ is a maximal subgroup of $End(\mathcal{A})$ containing S.

Let $\mathcal{I} = \{\text{ideal subgroups of } S(\mathcal{A})\}$
$\mathcal{K} = \{$ kernels of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$
$\mathcal{R} = \{$ images of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$

Fact

The mapping $\mathcal{I} \ni S \mapsto \langle K_S, R_S \rangle \in \mathcal{K} \times \mathcal{R}$ is a bijection $\mathcal{I} \to \mathcal{K} \times \mathcal{R}$.
The fibers of the surjective mapping $APer(S(\mathcal{A})) \ni p \mapsto Ker(d_p) \in \mathcal{K}$ are precisely the minimal subflows of $S(\mathcal{A})$.

Proposition

Assume that S is a subgroup of $End(\mathcal{A})$.Then all $f \in S$ have a common kernel $K = K_S$ and common image $R = R_S$.Let $S_{K,R} = \{f \in End(\mathcal{A}) : Ker(f) = K,\ Im(f) = R$ and $f|_R$ permutes $R\}$. Then $S_{K,R}$ is a maximal subgroup of $End(\mathcal{A})$ containing S.

Let $\mathcal{I} = \{$ideal subgroups of $S(\mathcal{A})\}$
$\mathcal{K} = \{$ kernels of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$
$\mathcal{R} = \{$ images of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$

Fact

The mapping $\mathcal{I} \ni S \mapsto \langle K_S, R_S \rangle \in \mathcal{K} \times \mathcal{R}$ is a bijection $\mathcal{I} \to \mathcal{K} \times \mathcal{R}$.
The fibers of the surjective mapping $APer(S(\mathcal{A})) \ni p \mapsto Ker(d_p) \in \mathcal{K}$ are precisely the minimal subflows of $S(\mathcal{A})$.

Subgroups of $E(S(\mathcal{A})) \cong S(\mathcal{A}) \cong End(\mathcal{A})$

Proposition

Assume that S is a subgroup of $End(\mathcal{A})$.Then all $f \in S$ have a common kernel $K = K_S$ and common image $R = R_S$.Let $S_{K,R} = \{f \in End(\mathcal{A}) : Ker(f) = K,\ Im(f) = R \text{ and } f|_R \text{ permutes } R\}$. Then $S_{K,R}$ is a maximal subgroup of $End(\mathcal{A})$ containing S.

Let $\mathcal{I} = \{\text{ideal subgroups of } S(\mathcal{A})\}$
$\mathcal{K} = \{$ kernels of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$
$\mathcal{R} = \{$ images of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$

Fact

The mapping $\mathcal{I} \ni S \mapsto \langle K_S, R_S \rangle \in \mathcal{K} \times \mathcal{R}$ is a bijection $\mathcal{I} \to \mathcal{K} \times \mathcal{R}$.
The fibers of the surjective mapping $APer(S(\mathcal{A})) \ni p \mapsto Ker(d_p) \in \mathcal{K}$ are precisely the minimal subflows of $S(\mathcal{A})$.

Subgroups of $E(S(\mathcal{A})) \cong S(\mathcal{A}) \cong End(\mathcal{A})$

Proposition

Assume that S is a subgroup of $End(\mathcal{A})$.Then all $f \in S$ have a common kernel $K = K_S$ and common image $R = R_S$.Let $S_{K,R} = \{f \in End(\mathcal{A}) : Ker(f) = K,\ Im(f) = R \text{ and } f|_R \text{ permutes } R\}$. Then $S_{K,R}$ is a maximal subgroup of $End(\mathcal{A})$ containing S.

Let $\mathcal{I} = \{\text{ideal subgroups of } S(\mathcal{A})\}$
$\mathcal{K} = \{$ kernels of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$
$\mathcal{R} = \{$ images of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$

Fact

The mapping $\mathcal{I} \ni S \mapsto \langle K_S, R_S \rangle \in \mathcal{K} \times \mathcal{R}$ is a bijection $\mathcal{I} \to \mathcal{K} \times \mathcal{R}$.
The fibers of the surjective mapping $APer(S(\mathcal{A})) \ni p \mapsto Ker(d_p) \in \mathcal{K}$ are precisely the minimal subflows of $S(\mathcal{A})$.

Subgroups of $E(S(\mathcal{A})) \cong S(\mathcal{A}) \cong End(\mathcal{A})$

Proposition

Assume that S is a subgroup of $End(\mathcal{A})$.Then all $f \in S$ have a common kernel $K = K_S$ and common image $R = R_S$.Let $S_{K,R} = \{f \in End(\mathcal{A}) : Ker(f) = K,\ Im(f) = R \text{ and } f|_R \text{ permutes } R\}$. Then $S_{K,R}$ is a maximal subgroup of $End(\mathcal{A})$ containing S.

Let $\mathcal{I} = \{$ideal subgroups of $S(\mathcal{A})\}$
$\mathcal{K} = \{$ kernels of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$
$\mathcal{R} = \{$ images of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$

Fact

The mapping $\mathcal{I} \ni S \mapsto \langle K_S, R_S\rangle \in \mathcal{K} \times \mathcal{R}$ is a bijection $\mathcal{I} \to \mathcal{K} \times \mathcal{R}$.
The fibers of the surjective mapping $APer(S(\mathcal{A})) \ni p \mapsto Ker(d_p) \in \mathcal{K}$ are precisely the minimal subflows of $S(\mathcal{A})$.

Proposition

Assume that S is a subgroup of $End(\mathcal{A})$.Then all $f \in S$ have a common kernel $K = K_S$ and common image $R = R_S$.Let $S_{K,R} = \{f \in End(\mathcal{A}) : Ker(f) = K,\ Im(f) = R \text{ and } f|_R \text{ permutes } R\}$. Then $S_{K,R}$ is a maximal subgroup of $End(\mathcal{A})$ containing S.

Let $\mathcal{I} = \{\text{ideal subgroups of } S(\mathcal{A})\}$
$\mathcal{K} = \{$ kernels of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$
$\mathcal{R} = \{$ images of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$

Fact

The mapping $\mathcal{I} \ni S \mapsto \langle K_S, R_S \rangle \in \mathcal{K} \times \mathcal{R}$ is a bijection $\mathcal{I} \to \mathcal{K} \times \mathcal{R}$.

The fibers of the surjective mapping $APer(S(\mathcal{A})) \ni p \mapsto Ker(d_p) \in \mathcal{K}$ are precisely the minimal subflows of $S(\mathcal{A})$.

Subgroups of $E(S(\mathcal{A})) \cong S(\mathcal{A}) \cong End(\mathcal{A})$

Proposition

Assume that S is a subgroup of $End(\mathcal{A})$.Then all $f \in S$ have a common kernel $K = K_S$ and common image $R = R_S$.Let $S_{K,R} = \{f \in End(\mathcal{A}) : Ker(f) = K,\ Im(f) = R \text{ and } f|_R \text{ permutes } R\}$. Then $S_{K,R}$ is a maximal subgroup of $End(\mathcal{A})$ containing S.

Let $\mathcal{I} = \{\text{ideal subgroups of } S(\mathcal{A})\}$
$\mathcal{K} = \{$ kernels of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$
$\mathcal{R} = \{$ images of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$

Fact

The mapping $\mathcal{I} \ni S \mapsto \langle K_S, R_S\rangle \in \mathcal{K} \times \mathcal{R}$ is a bijection $\mathcal{I} \to \mathcal{K} \times \mathcal{R}$.
The fibers of the surjective mapping $APer(S(\mathcal{A})) \ni p \mapsto Ker(d_p) \in \mathcal{K}$ are precisely the minimal subflows of $S(\mathcal{A})$.

Subgroups of $E(S(\mathcal{A})) \cong S(\mathcal{A}) \cong End(\mathcal{A})$

Proposition

Assume that S is a subgroup of $End(\mathcal{A})$.Then all $f \in S$ have a common kernel $K = K_S$ and common image $R = R_S$.Let $S_{K,R} = \{f \in End(\mathcal{A}) : Ker(f) = K,\ Im(f) = R$ and $f|_R$ permutes $R\}$. Then $S_{K,R}$ is a maximal subgroup of $End(\mathcal{A})$ containing S.

Let $\mathcal{I} = \{$ideal subgroups of $S(\mathcal{A})\}$
$\mathcal{K} = \{$ kernels of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$
$\mathcal{R} = \{$ images of ideal subgroups of $S(\mathcal{A}) \cong End(\mathcal{A})\}$

Fact

The mapping $\mathcal{I} \ni S \mapsto \langle K_S, R_S \rangle \in \mathcal{K} \times \mathcal{R}$ is a bijection $\mathcal{I} \to \mathcal{K} \times \mathcal{R}$.
The fibers of the surjective mapping $APer(S(\mathcal{A})) \ni p \mapsto Ker(d_p) \in \mathcal{K}$ are precisely the minimal subflows of $S(\mathcal{A})$.

Maximal subgroups of $S_G(M)$ and $S_{G,\Delta}(M)$

Example

Let $H < G$ be Δ-definable, Δ-connected (i.e. $Mlt_\Delta(H) = 1$)
(So: $\exists! p_H \in S_{G,\Delta}(M)$ generic of H.)
Let $N = N_G(H) < G$ and $S_{p_H} = \{n \cdot p_H : n \in N\} \subseteq S_{G,\Delta}(M)$.
Then S_{p_H} is a maximal subgroup of $S_{G,\Delta}(M)$ and $S_{p_H} \cong_{def} N/H$.

Theorem 3

1. All maximal subgroups of $S_{G,\Delta}(M)$ are of this form.
2. If $S \subseteq S_G(M)$ is a maximal subgroup, then $S = invlim_{\Delta \in Inv} S_\Delta$ for some maximal subgroups $S_\Delta \subseteq S_{G,\Delta}(M)$.

Example

Let $H < G$ be Δ-definable, Δ-connected (i.e. $Mlt_\Delta(H) = 1$)
(So: $\exists! p_H \in S_{G,\Delta}(M)$ generic of H.)
Let $N = N_G(H) < G$ and $S_{p_H} = \{n \cdot p_H : n \in N\} \subseteq S_{G,\Delta}(M)$.
Then S_{p_H} is a maximal subgroup of $S_{G,\Delta}(M)$ and $S_{p_H} \cong_{def} N/H$.

Theorem 3

1. All maximal subgroups of $S_{G,\Delta}(M)$ are of this form.
2. If $S \subseteq S_G(M)$ is a maximal subgroup, then $S = invlim_{\Delta \in Inv} S_\Delta$ for some maximal subgroups $S_\Delta \subseteq S_{G,\Delta}(M)$.

Maximal subgroups of $S_G(M)$ and $S_{G,\Delta}(M)$

Example

Let $H < G$ be Δ-definable, Δ-connected (i.e. $Mlt_\Delta(H) = 1$)
(So: $\exists! p_H \in S_{G,\Delta}(M)$ generic of H.)
Let $N = N_G(H) < G$ and $S_{p_H} = \{n \cdot p_H : n \in N\} \subseteq S_{G,\Delta}(M)$.
Then S_{p_H} is a maximal subgroup of $S_{G,\Delta}(M)$ and $S_{p_H} \cong_{def} N/H$.

Theorem 3

1. All maximal subgroups of $S_{G,\Delta}(M)$ are of this form.
2. If $S \subseteq S_G(M)$ is a maximal subgroup, then $S = invlim_{\Delta \in Inv} S_\Delta$ for some maximal subgroups $S_\Delta \subseteq S_{G,\Delta}(M)$.

Example

Let $H < G$ be Δ-definable, Δ-connected (i.e. $Mlt_\Delta(H) = 1$)
(So: $\exists ! p_H \in S_{G,\Delta}(M)$ generic of H.)
Let $N = N_G(H) < G$ and $S_{p_H} = \{n \cdot p_H : n \in N\} \subseteq S_{G,\Delta}(M)$.
Then S_{p_H} is a maximal subgroup of $S_{G,\Delta}(M)$ and $S_{p_H} \cong_{def} N/H$.

Theorem 3

1. All maximal subgroups of $S_{G,\Delta}(M)$ are of this form.
2. If $S \subseteq S_G(M)$ is a maximal subgroup, then $S = invlim_{\Delta \in Inv} S_\Delta$ for some maximal subgroups $S_\Delta \subseteq S_{G,\Delta}(M)$.

Maximal subgroups of $S_G(M)$ and $S_{G,\Delta}(M)$

Example

Let $H < G$ be Δ-definable, Δ-connected (i.e. $Mlt_\Delta(H) = 1$)
(So: $\exists! p_H \in S_{G,\Delta}(M)$ generic of H.)
Let $N = N_G(H) < G$ and $S_{p_H} = \{n \cdot p_H : n \in N\} \subseteq S_{G,\Delta}(M)$.
Then S_{p_H} is a maximal subgroup of $S_{G,\Delta}(M)$ and $S_{p_H} \cong_{def} N/H$.

Theorem 3

1. All maximal subgroups of $S_{G,\Delta}(M)$ are of this form.
2. If $S \subseteq S_G(M)$ is a maximal subgroup, then $S = invlim_{\Delta \in Inv} S_\Delta$ for some maximal subgroups $S_\Delta \subseteq S_{G,\Delta}(M)$.

Example

Let $H < G$ be Δ-definable, Δ-connected (i.e. $Mlt_\Delta(H) = 1$)
(So: $\exists! p_H \in S_{G,\Delta}(M)$ generic of H.)
Let $N = N_G(H) < G$ and $S_{p_H} = \{n \cdot p_H : n \in N\} \subseteq S_{G,\Delta}(M)$.
Then S_{p_H} is a maximal subgroup of $S_{G,\Delta}(M)$ and $S_{p_H} \cong_{def} N/H$.

Theorem 3

1. All maximal subgroups of $S_{G,\Delta}(M)$ are of this form.
2. If $S \subseteq S_G(M)$ is a maximal subgroup, then $S = invlim_{\Delta \in Inv} S_\Delta$ for some maximal subgroups $S_\Delta \subseteq S_{G,\Delta}(M)$.

Example

Let $H < G$ be Δ-definable, Δ-connected (i.e. $Mlt_\Delta(H) = 1$)
(So: $\exists! p_H \in S_{G,\Delta}(M)$ generic of H.)
Let $N = N_G(H) < G$ and $S_{p_H} = \{n \cdot p_H : n \in N\} \subseteq S_{G,\Delta}(M)$.
Then S_{p_H} is a maximal subgroup of $S_{G,\Delta}(M)$ and $S_{p_H} \cong_{def} N/H$.

Theorem 3

1. All maximal subgroups of $S_{G,\Delta}(M)$ are of this form.
2. If $S \subseteq S_G(M)$ is a maximal subgroup, then $S = invlim_{\Delta \in Inv} S_\Delta$ for some maximal subgroups $S_\Delta \subseteq S_{G,\Delta}(M)$.

Example

Let $H < G$ be Δ-definable, Δ-connected (i.e. $Mlt_\Delta(H) = 1$)
(So: $\exists! p_H \in S_{G,\Delta}(M)$ generic of H.)
Let $N = N_G(H) < G$ and $S_{p_H} = \{n \cdot p_H : n \in N\} \subseteq S_{G,\Delta}(M)$.
Then S_{p_H} is a maximal subgroup of $S_{G,\Delta}(M)$ and $S_{p_H} \cong_{def} N/H$.

Theorem 3

1. All maximal subgroups of $S_{G,\Delta}(M)$ are of this form.
2. If $S \subseteq S_G(M)$ is a maximal subgroup, then $S = invlim_{\Delta \in Inv} S_\Delta$ for some maximal subgroups $S_\Delta \subseteq S_{G,\Delta}(M)$.

Maximal subgroups of $S_G(M)$ and $S_{G,\Delta}(M)$

Example

Let $H < G$ be Δ-definable, Δ-connected (i.e. $Mlt_\Delta(H) = 1$)
(So: $\exists ! p_H \in S_{G,\Delta}(M)$ generic of H.)
Let $N = N_G(H) < G$ and $S_{p_H} = \{n \cdot p_H : n \in N\} \subseteq S_{G,\Delta}(M)$.
Then S_{p_H} is a maximal subgroup of $S_{G,\Delta}(M)$ and $S_{p_H} \cong_{def} N/H$.

Theorem 3

1. All maximal subgroups of $S_{G,\Delta}(M)$ are of this form.
2. If $S \subseteq S_G(M)$ is a maximal subgroup, then $S = invlim_{\Delta \in Inv} S_\Delta$ for some maximal subgroups $S_\Delta \subseteq S_{G,\Delta}(M)$.

Example

Let $H < G$ be Δ-definable, Δ-connected (i.e. $Mlt_\Delta(H) = 1$)
(So: $\exists! p_H \in S_{G,\Delta}(M)$ generic of H.)
Let $N = N_G(H) < G$ and $S_{p_H} = \{n \cdot p_H : n \in N\} \subseteq S_{G,\Delta}(M)$.
Then S_{p_H} is a maximal subgroup of $S_{G,\Delta}(M)$ and $S_{p_H} \cong_{def} N/H$.

Theorem 3

1. All maximal subgroups of $S_{G,\Delta}(M)$ are of this form.
2. If $S \subseteq S_G(M)$ is a maximal subgroup, then $S = invlim_{\Delta \in Inv} S_\Delta$ for some maximal subgroups $S_\Delta \subseteq S_{G,\Delta}(M)$.

Types as functions

$$S_G(M) \ni p \rightsquigarrow d_p : Def_G(M) \to Def_G(M)$$

$$S_{G,\Delta}(M) \ni p \rightsquigarrow d_p : Def_{G,\Delta}(M) \to Def_{G,\Delta}(M)$$

$$d_p \rightsquigarrow Ker(d_p),\ Im(d_p)$$

$$Ker(d_p) = \{U \in Def_{G,\Delta}(M) : [U] \cap cl(Gp) = \emptyset\}$$

Idea

- The larger the type $p \in S_G(M)$, $p \in S_{G,\Delta}(M)$
- The smaller the flow $cl(Gp)$.
- The larger the kernel $Ker(d_p)$.
- The smaller the image $Im(d_p)$.
- The larger the (local) Morley rank of p.

Types as functions

$$S_G(M) \ni p \rightsquigarrow d_p : Def_G(M) \to Def_G(M)$$

$$S_{G,\Delta}(M) \ni p \rightsquigarrow d_p : Def_{G,\Delta}(M) \to Def_{G,\Delta}(M)$$

$$d_p \rightsquigarrow Ker(d_p),\ Im(d_p)$$

$$Ker(d_p) = \{U \in Def_{G,\Delta}(M) : [U] \cap cl(Gp) = \emptyset\}$$

Idea

- The larger the type $p \in S_G(M),\ p \in S_{G,\Delta}(M)$
- The smaller the flow $cl(Gp)$.
- The larger the kernel $Ker(d_p)$.
- The smaller the image $Im(d_p)$.
- The larger the (local) Morley rank of p.

Types as functions

$$S_G(M) \ni p \rightsquigarrow d_p : Def_G(M) \to Def_G(M)$$

$$S_{G,\Delta}(M) \ni p \rightsquigarrow d_p : Def_{G,\Delta}(M) \to Def_{G,\Delta}(M)$$

$$d_p \rightsquigarrow Ker(d_p),\ Im(d_p)$$

$$Ker(d_p) = \{U \in Def_{G,\Delta}(M) : [U] \cap cl(Gp) = \emptyset\}$$

Idea

- The larger the type $p \in S_G(M),\ p \in S_{G,\Delta}(M)$
- The smaller the flow $cl(Gp)$.
- The larger the kernel $Ker(d_p)$.
- The smaller the image $Im(d_p)$.
- The larger the (local) Morley rank of p.

Types as functions

$$S_G(M) \ni p \rightsquigarrow d_p : Def_G(M) \to Def_G(M)$$

$$S_{G,\Delta}(M) \ni p \rightsquigarrow d_p : Def_{G,\Delta}(M) \to Def_{G,\Delta}(M)$$

$$d_p \rightsquigarrow Ker(d_p),\ Im(d_p)$$

$$Ker(d_p) = \{U \in Def_{G,\Delta}(M) : [U] \cap cl(Gp) = \emptyset\}$$

Idea

- The larger the type $p \in S_G(M),\ p \in S_{G,\Delta}(M)$
- The smaller the flow $cl(Gp)$.
- The larger the kernel $Ker(d_p)$.
- The smaller the image $Im(d_p)$.
- The larger the (local) Morley rank of p.

Types as functions

$$S_G(M) \ni p \rightsquigarrow d_p : Def_G(M) \to Def_G(M)$$

$$S_{G,\Delta}(M) \ni p \rightsquigarrow d_p : Def_{G,\Delta}(M) \to Def_{G,\Delta}(M)$$

$$d_p \rightsquigarrow Ker(d_p),\ Im(d_p)$$

$$Ker(d_p) = \{U \in Def_{G,\Delta}(M) : [U] \cap cl(Gp) = \emptyset\}$$

Idea

- The larger the type $p \in S_G(M),\ p \in S_{G,\Delta}(M)$
- The smaller the flow $cl(Gp)$.
- The larger the kernel $Ker(d_p)$.
- The smaller the image $Im(d_p)$.
- The larger the (local) Morley rank of p.

Types as functions

$$S_G(M) \ni p \rightsquigarrow d_p : Def_G(M) \to Def_G(M)$$

$$S_{G,\Delta}(M) \ni p \rightsquigarrow d_p : Def_{G,\Delta}(M) \to Def_{G,\Delta}(M)$$

$$d_p \rightsquigarrow Ker(d_p),\ Im(d_p)$$

$$Ker(d_p) = \{U \in Def_{G,\Delta}(M) : [U] \cap cl(Gp) = \emptyset\}$$

Idea

- The larger the type $p \in S_G(M)$, $p \in S_{G,\Delta}(M)$
- The smaller the flow $cl(Gp)$.
- The larger the kernel $Ker(d_p)$.
- The smaller the image $Im(d_p)$.
- The larger the (local) Morley rank of p.

Types as functions

$$S_G(M) \ni p \rightsquigarrow d_p : Def_G(M) \to Def_G(M)$$

$$S_{G,\Delta}(M) \ni p \rightsquigarrow d_p : Def_{G,\Delta}(M) \to Def_{G,\Delta}(M)$$

$$d_p \rightsquigarrow Ker(d_p),\ Im(d_p)$$

$$Ker(d_p) = \{U \in Def_{G,\Delta}(M) : [U] \cap cl(Gp) = \emptyset\}$$

Idea

- The larger the type $p \in S_G(M),\ p \in S_{G,\Delta}(M)$
- The smaller the flow $cl(Gp)$.
- The larger the kernel $Ker(d_p)$.
- The smaller the image $Im(d_p)$.
- The larger the (local) Morley rank of p.

Types as functions

$$S_G(M) \ni p \rightsquigarrow d_p : Def_G(M) \to Def_G(M)$$

$$S_{G,\Delta}(M) \ni p \rightsquigarrow d_p : Def_{G,\Delta}(M) \to Def_{G,\Delta}(M)$$

$$d_p \rightsquigarrow Ker(d_p),\ Im(d_p)$$

$$Ker(d_p) = \{U \in Def_{G,\Delta}(M) : [U] \cap cl(Gp) = \emptyset\}$$

Idea

- The larger the type $p \in S_G(M),\ p \in S_{G,\Delta}(M)$
- The smaller the flow $cl(Gp)$.
- The larger the kernel $Ker(d_p)$.
- The smaller the image $Im(d_p)$.
- The larger the (local) Morley rank of p.

Types as functions

$Ker(d_p), Im(d_p)$: measures of the size of p.

Let $p \in S_G(M)$ (or $p \in S_{G,\Delta}(M)$...)

Let $p^{*n} = \underbrace{p * \cdots * p}_{n}$.So $d_{p^{*n}} = \underbrace{d_p \circ \cdots \circ d_p}_{n}$.

Let $R(p) = \langle RM_\Delta(p) : \Delta \in Inv \rangle$.

Lemma

1. $R(p^{*n})$ grow (coordinatewise),$Ker(d_{p^{*n}})$ grow and $Im(d_{p^{*n}})$ shrink with $n = 1, 2, 3, \ldots$.
2. The growth/shrinking of these three sequences is strictly correlated.

Theorem 4

Let $p \in S_G(M)$. Then p is "profinitely many steps away" from a translate of a generic type of a connected type-definable subgroup of G.

Types as functions

$Ker(d_p), Im(d_p)$: measures of the size of p.

Let $p \in S_G(M)$ (or $p \in S_{G,\Delta}(M)$...)

Let $p^{*n} = \underbrace{p * \cdots * p}_{n}$.So $d_{p^{*n}} = \underbrace{d_p \circ \cdots \circ d_p}_{n}$.

Let $R(p) = \langle RM_\Delta(p) : \Delta \in Inv \rangle$.

Lemma

1. $R(p^{*n})$ grow (coordinatewise),$Ker(d_{p^{*n}})$ grow and $Im(d_{p^{*n}})$ shrink with $n = 1, 2, 3, \ldots$.
2. The growth/shrinking of these three sequences is strictly correlated.

Theorem 4

Let $p \in S_G(M)$. Then p is "profinitely many steps away" from a translate of a generic type of a connected type-definable subgroup of G.

Types as functions

$Ker(d_p), Im(d_p)$: measures of the size of p.
Let $p \in S_G(M)$ (or $p \in S_{G,\Delta}(M)$...)
Let $p^{*n} = \underbrace{p * \cdots * p}_{n}$. So $d_{p^{*n}} = \underbrace{d_p \circ \cdots \circ d_p}_{n}$.

Let $R(p) = \langle RM_\Delta(p) : \Delta \in Inv \rangle$.

Lemma

1. $R(p^{*n})$ grow (coordinatewise), $Ker(d_{p^{*n}})$ grow and $Im(d_{p^{*n}})$ shrink with $n = 1, 2, 3, \ldots$.
2. The growth/shrinking of these three sequences is strictly correlated.

Theorem 4

Let $p \in S_G(M)$. Then p is "profinitely many steps away" from a translate of a generic type of a connected type-definable subgroup of G.

Types as functions

$Ker(d_p), Im(d_p)$: measures of the size of p.
Let $p \in S_G(M)$ (or $p \in S_{G,\Delta}(M)$...)
Let $p^{*n} = \underbrace{p * \cdots * p}_{n}$. So $d_{p^{*n}} = \underbrace{d_p \circ \cdots \circ d_p}_{n}$.

Let $R(p) = \langle RM_\Delta(p) : \Delta \in Inv \rangle$.

Lemma

1. $R(p^{*n})$ grow (coordinatewise), $Ker(d_{p^{*n}})$ grow and $Im(d_{p^{*n}})$ shrink with $n = 1, 2, 3, \ldots$.
2. The growth/shrinking of these three sequences is strictly correlated.

Theorem 4

Let $p \in S_G(M)$. Then p is "profinitely many steps away" from a translate of a generic type of a connected type-definable subgroup of G.

Types as functions

$Ker(d_p), Im(d_p)$: measures of the size of p.
Let $p \in S_G(M)$ (or $p \in S_{G,\Delta}(M)$...)
Let $p^{*n} = \underbrace{p * \cdots * p}_{n}$.So $d_{p^{*n}} = \underbrace{d_p \circ \cdots \circ d_p}_{n}$.
Let $R(p) = \langle RM_\Delta(p) : \Delta \in Inv \rangle$.

Lemma

1. $R(p^{*n})$ grow (coordinatewise),$Ker(d_{p^{*n}})$ grow and $Im(d_{p^{*n}})$ shrink with $n = 1, 2, 3, \ldots$.
2. The growth/shrinking of these three sequences is strictly correlated.

Theorem 4

Let $p \in S_G(M)$. Then p is "profinitely many steps away" from a translate of a generic type of a connected type-definable subgroup of G.

$Ker(d_p), Im(d_p)$: measures of the size of p.
Let $p \in S_G(M)$ (or $p \in S_{G,\Delta}(M)$...)
Let $p^{*n} = \underbrace{p * \cdots * p}_{n}$. So $d_{p^{*n}} = \underbrace{d_p \circ \cdots \circ d_p}_{n}$.
Let $R(p) = \langle RM_\Delta(p) : \Delta \in Inv \rangle$.

Lemma

1. $R(p^{*n})$ grow (coordinatewise), $Ker(d_{p^{*n}})$ grow and $Im(d_{p^{*n}})$ shrink with $n = 1, 2, 3, \ldots$.
2. The growth/shrinking of these three sequences is strictly correlated.

Theorem 4

Let $p \in S_G(M)$. Then p is "profinitely many steps away" from a translate of a generic type of a connected type-definable subgroup of G.

Types as functions

$Ker(d_p), Im(d_p)$: measures of the size of p.
Let $p \in S_G(M)$ (or $p \in S_{G,\Delta}(M)$...)
Let $p^{*n} = \underbrace{p * \cdots * p}_{n}$.So $d_{p^{*n}} = \underbrace{d_p \circ \cdots \circ d_p}_{n}$.
Let $R(p) = \langle RM_\Delta(p) : \Delta \in Inv \rangle$.

Lemma

1. $R(p^{*n})$ grow (coordinatewise), $Ker(d_{p^{*n}})$ grow and $Im(d_{p^{*n}})$ shrink with $n = 1, 2, 3, \ldots$.
2. The growth/shrinking of these three sequences is strictly correlated.

Theorem 4

Let $p \in S_G(M)$. Then p is "profinitely many steps away" from a translate of a generic type of a connected type-definable subgroup of G.

Types as functions

$Ker(d_p), Im(d_p)$: measures of the size of p.
Let $p \in S_G(M)$ (or $p \in S_{G,\Delta}(M)$...)
Let $p^{*n} = \underbrace{p * \cdots * p}_{n}$. So $d_{p^{*n}} = \underbrace{d_p \circ \cdots \circ d_p}_{n}$.
Let $R(p) = \langle RM_\Delta(p) : \Delta \in Inv \rangle$.

Lemma

1. $R(p^{*n})$ grow (coordinatewise), $Ker(d_{p^{*n}})$ grow and $Im(d_{p^{*n}})$ shrink with $n = 1, 2, 3, \ldots$.
2. The growth/shrinking of these three sequences is strictly correlated.

Theorem 4

Let $p \in S_G(M)$. Then p is "profinitely many steps away" from a translate of a generic type of a connected type-definable subgroup of G.

Types as functions

$Ker(d_p), Im(d_p)$: measures of the size of p.
Let $p \in S_G(M)$ (or $p \in S_{G,\Delta}(M)$...)
Let $p^{*n} = \underbrace{p * \cdots * p}_{n}$.So $d_{p^{*n}} = \underbrace{d_p \circ \cdots \circ d_p}_{n}$.
Let $R(p) = \langle RM_\Delta(p) : \Delta \in Inv \rangle$.

Lemma

1. $R(p^{*n})$ grow (coordinatewise),$Ker(d_{p^{*n}})$ grow and $Im(d_{p^{*n}})$ shrink with $n = 1, 2, 3, \ldots$.
2. The growth/shrinking of these three sequences is strictly correlated.

Theorem 4

Let $p \in S_G(M)$. Then p is "profinitely many steps away" from a translate of a generic type of a connected type-definable subgroup of G.

Proof of Theorem 4

Let $\Delta \in Inv$, $n_\Delta = RM_\Delta(G)$, $p_\Delta = p|_\Delta \in S_{G,\Delta}(M)$. Then:

1. $p_\Delta^{*n_\Delta} \in$ a maximal subgroup S of $S_{G,\Delta}(M)$.
2. $p_\Delta^{*n_\Delta}$ is a translate of a generic type of a Δ-definable Δ-connected group $H < G$
3. For every $l \geq n_\Delta$, items 1. and 2. hold for p_Δ^{*l} in place of $p_\Delta^{*n_\Delta}$, with the same S and H.

Corollary

$S_{G,\Delta}(M)^{n_\Delta} = \bigcup\{\text{subgroups of } S_{G,\Delta}(M)\}$.

Proof of Theorem 4

Let $\Delta \in Inv$, $n_\Delta = RM_\Delta(G)$, $p_\Delta = p|_\Delta \in S_{G,\Delta}(M)$. Then:

1. $p_\Delta^{*n_\Delta} \in$ a maximal subgroup S of $S_{G,\Delta}(M)$.
2. $p_\Delta^{*n_\Delta}$ is a translate of a generic type of a Δ-definable Δ-connected group $H < G$
3. For every $l \geq n_\Delta$, items 1. and 2. hold for p_Δ^{*l} in place of $p_\Delta^{*n_\Delta}$, with the same S and H.

Corollary

$S_{G,\Delta}(M)^{n_\Delta} = \bigcup\{\text{subgroups of } S_{G,\Delta}(M)\}$.

Proof of Theorem 4

Let $\Delta \in Inv$, $n_\Delta = RM_\Delta(G)$, $p_\Delta = p|_\Delta \in S_{G,\Delta}(M)$. Then:

1. $p_\Delta^{*n_\Delta} \in$ a maximal subgroup S of $S_{G,\Delta}(M)$.
2. $p_\Delta^{*n_\Delta}$ is a translate of a generic type of a Δ-definable Δ-connected group $H < G$
3. For every $l \geq n_\Delta$, items 1. and 2. hold for p_Δ^{*l} in place of $p_\Delta^{*n_\Delta}$, with the same S and H.

Corollary

$S_{G,\Delta}(M)^{n_\Delta} = \bigcup\{\text{subgroups of } S_{G,\Delta}(M)\}$.

Proof of Theorem 4

Let $\Delta \in Inv$, $n_\Delta = RM_\Delta(G)$, $p_\Delta = p|_\Delta \in S_{G,\Delta}(M)$. Then:

1. $p_\Delta^{*n_\Delta} \in$ a maximal subgroup S of $S_{G,\Delta}(M)$.
2. $p_\Delta^{*n_\Delta}$ is a translate of a generic type of a Δ-definable Δ-connected group $H < G$
3. For every $l \geq n_\Delta$, items 1. and 2. hold for p_Δ^{*l} in place of $p_\Delta^{*n_\Delta}$, with the same S and H.

Corollary

$S_{G,\Delta}(M)^{n_\Delta} = \bigcup\{\text{subgroups of } S_{G,\Delta}(M)\}$.

Proof of Theorem 4

Let $\Delta \in Inv$, $n_\Delta = RM_\Delta(G)$, $p_\Delta = p|_\Delta \in S_{G,\Delta}(M)$. Then:

1. $p_\Delta^{*n_\Delta} \in$ a maximal subgroup S of $S_{G,\Delta}(M)$.
2. $p_\Delta^{*n_\Delta}$ is a translate of a generic type of a Δ-definable Δ-connected group $H < G$
3. For every $l \geq n_\Delta$, items 1. and 2. hold for p_Δ^{*l} in place of $p_\Delta^{*n_\Delta}$, with the same S and H.

Corollary

$S_{G,\Delta}(M)^{n_\Delta} = \bigcup\{\text{subgroups of } S_{G,\Delta}(M)\}$.