Model Theory 2013

Ravello, 10th–15th June 2013

Schedule

Monday		Thursday	
9:30 – 9:45	Opening	9:15 – 10:15	Scanlon
9:45 – 10:45	Hils	10:15 – 10:45	Coffee Break
10:45 – 11:15	Coffee Break	10:45 – 11:45	Speissegger
11:15 – 11:45	Jones		
11:50 – 12:50	Sklinos	11:50 – 12:50	Sklinos
Lunch Break		Lunch Break	
16:00 – 16:30	Tea Break	16:00 – 16:30	Tea Break
16:30 – 17:30	Hart	16:30 – 17:00	Chernikov
17:40 – 18:40	Chatzidakis	17:04 – 18:05	Aschenbrenner
		18:15 – 18:45	Ugurlu
		Social Dinner	

Tuesday		Friday	
9:15 – 10:15	Sklinos	9:15 – 10:15	Scanlon
10:15 – 10:45	Coffee Break	10:15 – 10:45	Coffee Break
10:45 – 11:45	Breuillard	10:45 – 11:45	Peterzil
11:50 – 12:50	Hils	11:50 – 12:50	Habegger
Lunch Break		Lunch Break	
16:00 – 16:30	Tea Break		
16:30 – 17:30	Sela	17:00 – 17:30	Tea Break
17:40 – 18:40	Pillay	17:30 – 18:00	Towsner
		18:05 – 18:35	Conversano

Wednesday		Saturday	
9:15 – 10:15	Hils	9:15 – 10:15	Newelski
10:15 – 10:45	Coffee Break	10:15 – 10:45	Coffee Break
10:45 – 11:15	Malliaris	10:45 – 11:15	Fornasiero
11:20 – 12:20	Scanlon	11:20 – 12:20	Macpherson
Free Afternoon			

Talks

Author: **Matthias Aschenbrenner**
Title: **Gaps in H-Fields**

Abstract: This will be a continuation of my talk in Ravello, 2002. I will report on what we have learned about gaps in H-fields in the meantime.

Author: **Emmanuel Breuillard**
Title: **Height gap versus spectral gap**

Abstract: I will describe several instances in which height lower bounds are intimately related to spectral gaps for group representations. A Bogomolov type property (i.e. uniform height lower bound) on character varieties of reductive groups yields strengthenings of the Tits alternative and new uniform lower bounds for the spectral gaps of various group actions (cf. expander graphs). Conversely the recent advances on approximate groups give new spectral gap bounds for finite simple groups, which can be used to get new instances of the Bogomolov property.

Author: **Zoe Chatzidakis**
Title: **Around the Canonical Base Property**

Abstract: The Canonical Base Property (CBP for short) was introduced by Pillay, inspired by results in Complex geometry. It implies the dichotomy "modular/fields" for rank 1 types, and is satisfied by the two main examples of fields with operators.

After giving the definition, I will discuss how one can prove it, some of its consequences, and mention some applications to algebraic problems.

Author: **Artem Chernikov**
Title: **External definability and groups in NIP**

Abstract: NIP is a large class of first-order theories containing stable and o-minimal theories, along with algebraically closed valued fields and p-adics. Study of groups definable in NIP theories and their model theoretic connected components led to a resolution of Pillay's conjecture on o-minimal groups. While definability of types is a characteristic feature of stable theories, it turns out that externally definable sets still demonstrate tame behaviour in the general case of NIP theories. In this talk I will present some results demonstrating that under the NIP assumption, adding externally definable sets neither effects definable (extreme) amenability of a group, nor it changes any of its connected components G^0, G^{00}, G^{000}. These results have some applications to definable topological dynamics, in particular to the conjecture relating the Ellis group and G/G^{00} in the case of definably amenable groups in o-minimal theories. Joint work with Anand Pillay and Pierre Simon.

Author: **Annalisa Conversano**
Title: **A non-compact version of Pillay's conjecture**

Abstract: A beautiful conjecture of Pillay (now fully proved) connects groups definable in o-minimal structures with compact real Lie groups. We present a generalization of Pillay's conjecture to the non-compact case. That is, we discuss how a real Lie group can be associated in a canonical way to a group definable in an o-minimal structure, so that algebraic and topological properties (such as dimension, torsion, homotopy type...) are preserved.

Author: **Antongiulio Fornasiero**
Title: **A fundamental dichotomy for definably complete expansions of ordered fields**

Abstract: (Joint work with P. Hieronymi). An ordered structure K is definably complete (DC) if every definable subset of K has a least upper bound in $K \cup \{\pm\infty\}$. We extend P. Hieronymi's dichotomy from $\mathbb{R}$ to any DC field: an expansion of a DC field either defines a (unique) discrete subring N, or the image of any definable discrete set under any definable map is nowhere dense.

I the first case, one can encode definable subsets of N as elements of K, making N a model of second order Peano arithmetic; in the second case, K presents a "tame" behaviour (e.g.: every definable continuous function is differentiable outside a nowhere dense set). By distinguishing the two cases, one can transfer several theorems from $\mathbb{R}$ to K. Notable examples are Baire's Category Theorem and Lebesgue's Theorem on the differentiability of a monotone function.

Author: **Philipp Habegger**
Title: **Divisible Points on Curves**

Abstract: This is joint work with Martin Bays. Suppose C is an algebraic curve in a multiplicative torus. Serge Lang provided a sufficient condition for C to contain only finitely many points whose coordinates are in fixed, finitely generated multiplicative group. A related problem concerns the set of points x on C such that x^k with k at least 2 also lies on C. If the ambient dimension is at most 2, heuristics suggest that there are infinitely many such x. If the dimension is at least 3 then we are in the realm of Zilber's Conjecture on Intersections with Tori. Here we can prove finiteness in many relevant cases. Our approach is related to a strategy first proposed in another context by Zannier. It relies on a result of Pila and Wilkie on the distribution of rational points on a set definable in an o-minimal structure. In addition we use results from transcendence theory due to Baker and Rémond.

Author: **Bradd Hart**
Title: **Continuous model theory and the classification problem**

Abstract: There is a growing body of evidence that suggests that the model theory of operator algebras could say something about the Elliott programme and the classification of separable, simple nuclear algebras. I will explain the necessary background for both the model theory and operator algebras and give some indication of the state the art. This is joint work with Ilijas Farah.

AUTHOR: **Gareth Jones**
TITLE: **Local definability theory for holomoprhic functions**

ABSTRACT: Suppose that F is a collection of holomorphic functions containing all polynomials in any number of variables. Some time ago Wilkie asked for a characterization, in terms of natural analytic operations, of the holomorphic functions that are locally definable in the expansion of the real field by all restrictions to compact boxes of functions in F. He conjectured that if the class F is closed under differentiation and Schwarz reflection, then any function locally definable from F can be obtained from F by composition and extraction of implicit functions.

Jonathan Kirby, Tamara Servi and I have answered Wilkie's question, but not quite proved his conjecture. We show that if, in addition to the operations above, we are allowed monomial division then we obtain all locally definable functions. I'll explain the setting for all of this, and then sketch some parts of our proof.

AUTHOR: **H. Dugald MacPherson**
TITLE: **Countable structures with few reducts**

ABSTRACT: For a countably infinite structure M which is omega-categorical, we may define a reduct of M to be a structure M' with the same domain as M whose 0-definable relations are 0-definable in M, or equivalently, whose automorphism group $\mathrm{Aut}(M')$ contains $\mathrm{Aut}(M)$. For a fairly small class of omega-categorical structures, the reducts are fully described (work of Cameron, Thomas, and others).

If M is not omega-categorical, the two notions of reduct diverge. I will discuss recent work with Manuel Bodirsky, investigating examples of countably infinite structures which are not omega-categorical but have few or no proper non-trivial reducts, in both senses.

AUTHOR: **Maryanthe Malliaris**
TITLE: **On cofinality spectrum theorems**

ABSTRACT: Recently, Malliaris and Shelah resolved two a priori unrelated open problems, the question from model theory of whether SOP2 is maximal in Keisler's order and the question from set theory/general topology of whether "p = t", by showing that both can be translated into instances of a fundamental problem which could be stated and solved using model-theoretic methods. The talk will be about this common context of "cofinality spectrum problems" and its applications.

AUTHOR: **Julien Melleray**
TITLE: **Continuous logic, Polish groups, and graded sets**

ABSTRACT: Given a countable first-order structure, one can consider its automorphism group, which is a closed subgroup of the group of permutations of a countable set, hence a Polish group. The structure completely defines the group; the group remembers things (sometimes, everything) about the structure; this has led to an interesting interplay between model theory and descriptive set

theory (mostly via the use of Baire category methods). Using classical model theory, one can only discuss permutation groups; continuous logic enables one to broach the whole class of Polish groups, and I will discuss examples where this approach leads to useful definitions and concepts. As time allows, I will also discuss a "graded approach" to (part of) descriptive set theory, reminiscent of the move from model theory to continuous logic. This is joint work with Itaï Ben Yaacov.

AUTHOR: **Ludomir Newelski**
TITLE: **Topological dynamics of stable groups**

ABSTRACT: We show that the Ellis semigroup of a stable group G is an inverse limit of a system of type-definable semigroups. The crucial point in the proof is presentation of G-types as functions – endomorphisms of a G-algebra of definable subsets of G. Identifying types with functions yields a new way to measure types by the size of the kernel and the image of the associated endomorphisms. We discuss how this way is related to (local) Morley ranks.

AUTHOR: **Yaacov Peterzil**
TITLE: **Locally definable groups and lattices**

ABSTRACT: A locally definable group is a group G which is a given as a (countable) union of definable sets, with the group operation definable on each definable subset of $G \times G$. If in addition G is generated as a group by a definable subet then it is called definably generated.

It was conjectured in a joint paper with Eleftheriou that in o-minimal structures, every definably generated abelian group contains a definable generic set (i.e. a definable set such that boundedly many translates cover G). While the conjecture is still open, I will discuss several equivalent properties of G, as appear in a paper by Edmundo, Berarducci and Mamino, as well as one simple case in which the conjecture is solved (joint work with Eleftheriou).

AUTHOR: **Anand Pillay**
TITLE: **Galois theories, transcendence, Manin maps, and the model theory of differentially closed fields**

ABSTRACT: (joint with D. Bertrand) The results concern Ax-Schanuel type transcendence statements for families of (semi) abelian varieties, improving some earlier results, as well as the the injectivity of the Manin map when tensored with $\mathbb{C}$. Differential Galois theories (definable automorphism groups in DCF_0) play an important role.

AUTHOR: **Zlil Sela**
TITLE: **Varieties and geometric structures in semigroups**

ABSTRACT: In 1946 Quine proved that arithmetic can be interpreted in the theory of a free semigroup. Durnev and others proved that fragments of that theory (including the AE theory) are undecidable.

In a different direction, Makanin showed in 1977 that it is possible to decide if a system of equations over a free semigroup has a solution. This work preceeded his own work on the similar question for free groups. In 1987 Razborov managed to use Makanin's work on groups, and encoded the set of solutions to a system of equations over a free group in some combinatorial structures. No analogue of Razborov's work is known for varieties over a free semigroup.

We suggest a geometric approach to study varieties over a free semigroup. We manage to find analogues of (geometric) structures that were known to exist over groups to encode the points in a variety over a free semigroup.

AUTHOR: **Patrick Speissegger**
TITLE: **Pfaffian functions vs. Rolle leaves**

ABSTRACT: In the early 1980s, after Khovanskii's ICM lecture, van den Dries formulated the conjecture that the expansion P of the real field by all pfaffian functions was model complete. Thinking about the problem led him to formulate a minimality notion in expansions of the real order, which directly inspired Pillay and Steinhorn in their discovery of o-minimality. However, while P has been known to be o-minimal since Wilkie's groundbreaking work in 1996, van den Dries's conjecture is still open today. Recently, Lion and I proved a variant of this conjecture, in which "pfaffian functions" are replaced with "nested Rolle leaves", which in essence correspond to the objects originally studied by Khovanskii. The mystery lies in how these two expansions are related. I will explain each of them and exhibit a third related notion, found recently in joint work with Jones, which might help explain some of the mystery around van den Dries's conjecture.

AUTHOR: **Henry Towsner**
TITLE: **Pseudo-random graphs as structures with good enough measures**

ABSTRACT: An area of active interest in extremal graph theory is the extension of results about dense graphs (graphs in which some fixed positive fraction of possible edges are actually present) to graphs which are sparse but have various randomness properties—in particular, to graphs which are dense subgraphs of sufficiently pseudo-random sparse graphs. Many of the original versions of results in this area (for instance, Szemeredi regularity and triangle removal) have model-theoretic proofs which reveal a close relationship to the notion of stability. The model-theoretic approach generally proceeds through an ultraproduct in which there is a well-behaved measure; in the presence of a well-behaved measure, the model theory simplifies somewhat (for instance, there is a natural way to divide an arbitrary binary relation into a stable part and a "random" part uniquely—but only up to measure 0).

Attempting to do the same in the sparse setting creates a new obstacle: the measure is no longer well-behaved. Fortunately, the analogy holds up—pseudo-randomness in the finitary world corresponds to good behavior of the measure in the ultraproduct. Furthermore, the infinitary setting eliminates many detailed calculations, making it possible to prove new purely combinatorial results.

AUTHOR: **Pinar Ugurlu**
TITLE: **Linear Pseudofinite Groups**

ABSTRACT: Pseudofinite groups are infinite models of the theory of finite groups. Simple ones are classified by John S. Wilson as (twisted) Chevalley groups over pseudofinite fields (CFSG is used). Wilson's proof can be used to classify non-abelian definably simple pseudofinite groups of finite centralizer dimension, in particular the linear ones. We will see that this result yields an alternative proof of the Larsen-Pink Theorem. This theorem roughly says that "large" finite simple groups of matrices are Chevalley groups over finite fields. We will also talk about the ongoing project (suggested by A. Borovik) of eliminating CFSG from our classification proof which leads us to a possible 2-Sylow theory for pseudofinite groups.

Tutorials

AUTHOR: **Martin Hils**

TITLE: **Tameness in non-archimedean geometry through model theory**

ABSTRACT: If K is a (complete) field with respect to a non-archimedean absolute value, then K is totally disconnected as a topological field. This is a serious obstacle when one wants to develop analytic geometry over K as this is done in the complex-analytic case. One approach to overcome this problem is due to Berkovich in the late 80's. He develops a theory of K-analytic spaces, 'adding points' to the set of naive points. In particular, for an algebraic variety V defined over K he constructs what is now called its Berkovich analytification V^{an} which contains $V(K)$ (with its natural topology) as a subspace and which is locally compact and locally arcwise connected.

Recently, using advanced methods from the model theory of algebraically closed valued fields (ACVF), Hrushovski and Loeser considerably improved existing topological tameness results for the Berkovich analytifications of algebraic varieties, showing for example that these are always homotopic to a polytope and locally contractible, without any smoothness assumption on the variety.

More precisely, given an algebraic variety V over a valued field, they use the theory of stable domination in ACVF (due to Haskell-Hrushovski-Macpherson) to construct a (pro-)definable space $\hat{V}$ which is a close analogue of V^{an}, and they establish topological tameness properties in this definable setting, combining *o*-minimality with tools from stability theory. These properties are then shown to transfer to the actual Berkovich setting.

In the tutorial, we will give an introduction to this new approach to non-archimedean geometry. We will first recall some key results about ACVF which are used in the work of Hrushovski and Loeser, in particular around stable domination. We will then describe the construction of $\hat{V}$, with a particular emphasis on definability issues. The proof of the main tameness result of Hrushovski-Loeser, namely the existence of a definable strong deformation restraction of $\hat{V}$ onto a definable subspace which is internal to the value group (i.e. a piecewise linear object in the definable category) is quite involved. We will present the argument in the case where V is an algebraic curve, only giving a sketch of the proof in the higher dimensional case. Finally, we will indicate how one may transfer the results from $\hat{V}$ to V^{an}.

AUTHOR: **Rizos Sklinos**

TITLE: **Merzlyakov-type theorems after Sela**

ABSTRACT: In 1946 A.Tarski asked whether non abelian free groups share the same common first-order theory. The question remained open for more than 50 years, until Sela and independently Kharlampovich - Myasnikov gave a positive solution. Although Sela's proof was available since 2001 very little progress has been made towards the understanding of the definable sets in this new theory. The main reason is that the proof is very long and the arguments involved are hard to understand by both communities: model theorists and geometers.

The positive solution goes through a result of quantifier elimination down to boolean combinations of $\forall\exists$ formulas. Let us note that a (not correct) proof

of the equality of the $\forall\exists$ theories was known before Sela's result (see [Sac73]). Still we are far from understanding the "basic" sets of the theory of non abelian free groups.

The main intuitive idea that led to the above mentioned quantifier elimination is the technique of "formal solutions" (although the technical complications of the proof need numerous new ideas). The prototype of such results is the following old theorem by Merzlyakov [Mer66].

Theorem 1 *Let $\Sigma(\bar{x}, \bar{y})$ be a finite set of words in $\langle \bar{x}, \bar{y} \rangle$. Let $\mathbb{F}$ be a non abelian free group. Suppose $\mathbb{F} \models \forall \bar{x} \exists \bar{y} (\Sigma(\bar{x}, \bar{y}) = 1)$. Then there exists a retract $r : G_\Sigma \twoheadrightarrow \langle \bar{x} \rangle$, where $G_\Sigma := \langle \bar{x}, \bar{y} \mid \Sigma(\bar{x}, \bar{y}) \rangle$.*

In this tutorial we will present the technique of "formal solutions" together with applications concerning model theoretic questions. Our approach will be geometric following Sela's first two papers on the series of seven papers ([Sel01, Sel03, Sel05a, Sel04, Sel05b, Sel06a, Sel06b, Selb]) culminating to the full proof of the positive solution to Tarski's question.

We will start by presenting the geometric background needed, i.e. real trees, group actions on real trees and Rips' machine (see [Bes02, BF95] for some reference). This will naturally lead us to the definition of limit groups. Limit groups played a significant role in almost every step of Sela's proof. By our approach it will be apparent that limit groups are objects of geometry as they admit natural actions on real trees and many useful properties can be deduced from the action.

We will continue with the proof of Merzlyakov's theorem (in a slightly extended form). We will not follow Merzlyakov's combinatorial proof but rather Sela's geometric one. The main reason is that Sela's approach suggests some natural generalizations of Merzlyakov's theorem. The most essential tool in our presentation will be Sela's shortening argument, that we will intuitively explain and only prove some special cases.

We will be interested in groups G_R with presentation $\langle \bar{x} \mid R(\bar{x}) \rangle$ so that the following statement holds:

Statement 1 *Let $\Sigma(\bar{x}, \bar{y})$ be a finite set of words in $\langle \bar{x}, \bar{y} \rangle$. Let $\mathbb{F}$ be a non abelian free group. Suppose $\mathbb{F} \models \forall \bar{x} (R(\bar{x}) = 1 \to \exists \bar{y} (\Sigma(\bar{x}, \bar{y}) = 1))$. Then there exists a retract $r : G_\Sigma \twoheadrightarrow G_R$, where $G_\Sigma := \langle \bar{x}, \bar{y} \mid \Sigma(\bar{x}, \bar{y}) \rangle$.*

It turns out that groups that admit a special structure (that is called a *tower* in Sela's terminology) satisfy (some refined form) of the above statement. Groups that admit a tower structure form a sub-class of the class of limit groups. More notably, groups that admit a "hyperbolic" tower structure are exactly the finitely generated groups that satisfy the theory of non abelian free groups. We will see that a notion dominating our proofs will be the notion of a "test sequence". Test sequences can be thought of as "generic" elements (not in the sense of model theory) corresponding to a variety.

Finally, we will explain Sela's Diophantine envelopes corresponding to definable sets (see [Sela]). A Diophantine envelope can be thought of as a union of Diophantine sets that is very "close" to the original definable set in the sense that "generic" elements of the envelope live eventually in the set. We will combine our understanding of "test sequences" together with Sela's Diophantine envelopes in order to give some intuition around definability/non-definability results in the free group.

References

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AUTHOR: **Thomas Scanlon**

TITLE: **The Zilber-Pink Conjecture:**
Diophantine geometry, functional transcendence, differential algebra and o-minimality
A tutorial in three lectures

ABSTRACT: The collection of conjectures falling under the rubric of the Zilber-Pink conjecture amalgamate many difficult diophantine geometric problems including the André-Oort conjecture, the Mordell-Lang conjecture, Zilber's conjecture on intersection with tori, and the Bombieri-Masser-Zannier conjectures on anomalous intersections amongst others. As such, this conjecture codifies the principle that algebraic relations on special points or special varieties must be explained by geometric special relations. Model theoretically, the conjecture arose from Zilber's attempt to axiomatize the theory of $\mathbb{C}_{\exp} := (\mathbb{C}, +, \cdot, \exp)$ and has shown itself to be amenable to model theoretic treatments. (There have been important strictly number theoretic contributions, too, notably through the study of heights and Galois theory, but these lectures will focus on the model theoretic developments.)

In this lecture series, I will discuss the connections between the Zilber-Pink conjecture and the theory of $\mathbb{C}_{\exp}$, Ax's differential algebraic proof of the functional Schanuel conjecture and the consequences of Ax's theorem for the Zilber-Pink conjecture, generalizations of Ax's theorem for abelian exponentials, Pink's formulation of the conjecture in terms of mixed Shimura varieties, o-minimal methods towards Zilber-Pink and Ax-Lindemann-Weierstrass, and some curious applications to diophantine geometry and to the model theory of differential fields.