Weakly Projective MV-algebras

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- Introduction and motivation
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- Characterization of Weakly Projective MV-algebras
- Connections with admissibility

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Finitely Presented MV-algebras and Rational Polyhedra

Theorem (McNaughton (1951))

For each n = 1, 2, ..., the MV-algebra $\mathfrak{M}([0, 1]^n)$ of McNaughton maps from the n-cube is freely generated by the projection maps $\xi_i(a_1, ..., a_n) = a_i$.

A McNaughton map is a continuous function

 $f: [0,1]^n \to [0,1]$ satisfying:

There are linear polynomials p_1, \ldots, p_m with integer coefficients, such that for all $x \in [0, 1]^n$ there is $i \in \{1, \ldots, m\}$ with $f(x) = p_i(x)$.

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An MV-algebra A is **finitely presented** if there exist n and $f \in \mathcal{M}([0,1]^n)$ such that

 $A \cong \mathcal{M}([0,1]^n)/\text{cong}(f,1)$

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An MV-algebra A is **finitely presented** if there exist n and $f \in \mathcal{M}([0,1]^n)$ such that

$$A \cong \mathcal{M}([0,1]^n) / \operatorname{cong}(f,1) \cong \mathcal{M}([0,1]^n) \upharpoonright f^{-1}(1)$$

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Finitely Presented MV-algebras and Rational Polyhedra

A **rational polyhedron** P in $[0, 1]^n$ is a finite union of closed simplexes $P = S_1 \cup \cdots \cup S_t$ in $[0, 1]^n$ such that the coordinates of the vertices of every simplex S_i are rational numbers.

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Finitely Presented MV-algebras and Rational Polyhedra

A **rational polyhedron** P in $[0,1]^n$ is a finite union of closed simplexes $P = S_1 \cup \cdots \cup S_t$ in $[0,1]^n$ such that the coordinates of the vertices of every simplex S_i are rational numbers.

Theorem

A subset $P \subseteq [0,1]^n$ is a rational polyhedron if and only if it there exists $f \in \mathcal{M}([0,1]^n)$ such that $P = f^{-1}(1)$.

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Finitely Presented MV-algebras and Rational Polyhedra

An MV-algebra A is finitely presented if there exist n and a rational polyhedron $P \subseteq [0, 1]^n$ such that

 $A \cong \mathcal{M}(P)$

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Homomorphisms and \mathbb{Z} -maps

$$\mathcal{M}(P \subseteq [0,1]^n) \xrightarrow{h} \mathcal{M}(Q \subseteq [0,1]^m)$$
$$\xi_i \longmapsto h(\xi_i) \colon Q \to [0,1]$$

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Homomorphisms and Z-maps

Homomorphisms and \mathbb{Z} -maps

$$\mathcal{M}(P \subseteq [0,1]^n) \xrightarrow{h} \mathcal{M}(Q \subseteq [0,1]^m)$$

 $\xi_i \longmapsto h(\xi_i) \colon Q \to [0,1]$

$$\eta_h = (h(\xi_1), \dots, h(\xi_n)) \colon Q \to [0, 1]^n$$

$$P \stackrel{\eta_h}{\longleftarrow} Q$$

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Homomorphisms and \mathbb{Z} -maps

$$\mathcal{M}(P \subseteq [0,1]^n) \xrightarrow{h} \mathcal{M}(Q \subseteq [0,1]^m)$$

 $\xi_i \longmapsto h(\xi_i) \colon Q \to [0,1]$

$$\eta_h = (h(\xi_1), \dots, h(\xi_n)) \colon Q \to [0, 1]^n$$

$$P \stackrel{\eta_h}{\longleftarrow} Q$$

$$h(f) = f \circ \eta_h$$

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Homomorphisms and \mathbb{Z} -maps

Definition

Given rational polyhedra $P \subseteq [0, 1]^n$ and $Q \subseteq [0, 1]^m$ a continuous map $\eta \colon P \to Q$ is called a \mathbb{Z} -map if there are finite affine linear maps q_1, \ldots, q_k such that for each $x \in P$, $\eta(x) = q_i(x)$ for some $i = 1, \ldots, k$.

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Homomorphisms and \mathbb{Z} -maps

Definition

Given rational polyhedra $P \subseteq [0,1]^n$ and $Q \subseteq [0,1]^m$ a continuous map $\eta \colon P \to Q$ is called a **Z-map** if there are finite affine linear maps q_1, \ldots, q_k such that for each $x \in P$, $\eta(x) = q_i(x)$ for some $i = 1, \ldots, k$.

Lemma

 η is a \mathbb{Z} -map if and only if $\xi_i \circ \eta \in \mathfrak{M}(P)$ for each $i = 1, \dots, m$.

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Duality

The category of Rational Polyhedra with \mathbb{Z} -maps is dually equivalent to the category of finitely presented MV-algebras.

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Duality: Projectives

- Projectives are retractions of Free algebras.
- ▶ Retractions are preserved under dualities.

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Duality: Projectives

- Projectives are retractions of Free algebras.
- Retractions are preserved under dualities.

Theorem

A finitely generated MV-algebra is projective iff there exist a number $n=1,2,\ldots$ and a \mathbb{Z} -map $\eta\colon [0,1]^n\to [0,1]^n$ such that

- (i) $\eta \circ \eta = \eta$,
- (ii) $A \cong \mathcal{M}(\eta([0,1]^n))$.

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[LMC & D. Mundici, Projective MV-algebras and rational polyhedra, *Algebra Universalis* Volume 62, Number 1, 63-74]

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[LMC & D. Mundici, Projective MV-algebras and rational polyhedra, *Algebra Universalis* Volume 62, Number 1, 63-74]

[LMC & D. Mundici, Rational polyhedra and projective lattice-ordered abelian groups with order unit, *Communications in Contemporary Mathematics* (to appear)]

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Motivation

Can we characterize the range of \mathbb{Z} -maps which domain is some n-cube?

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Motivation

Can we characterize the range of \mathbb{Z} -maps which domain is some *n*-cube?

In other words:

Are there intrinsic necessary and sufficient conditions for a rational polyhedron P to be equal to $\eta([0,1]^n)$ for some *n* and some \mathbb{Z} -map η ?

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Analizing Z-maps

Lemma

Given rational polyhedra $P \subseteq [0,1]^n$ and $Q \subseteq [0,1]^m$ a map $\eta \colon P \to Q$ is a \mathbb{Z} -map iff there is a triangulation Δ of P such that over every simplex T of Δ , η coincides with a (affine) linear map η_T with integer coefficients.

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Analizing Z-maps

Given a *t*-simplex $S = \text{conv}(v_0, \dots, v_t) \subseteq [0, 1]^n$ then linear maps from S into $[0, 1]^m$ are in one-one correspondence with maps from $\{v_0, \dots, v_t\}$ into $[0, 1]^m$.

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Analizing Z-maps

Let $f: [0,1]^n \to \mathbb{R}^m$ be linear map with integer coefficients, i.e. there exists $M \in \mathbb{Z}^{n \times m}$ and $b \in \mathbb{Z}^m$ such that f(w) = Mw + b for each $w \in [0,1]^n$

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Analizing Z-maps

Let $f: [0,1]^n \to \mathbb{R}^m$ be linear map with integer coefficients, i.e. there exists $M \in \mathbb{Z}^{n \times m}$ and $b \in \mathbb{Z}^m$ such that f(w) = Mw + b for each $w \in [0,1]^n$

Let $v = (x_1, \dots, x_n) \in [0, 1]^n$ be a rational vector. We define $\operatorname{den}(v)$ to be the least common denominator of $\{x_1, \dots, x_n\}$, i.e. the smallest $k \in \mathbb{Z}$ such that $kv \in \mathbb{Z}^n$.

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Analizing Z-maps

Let $f: [0,1]^n \to \mathbb{R}^m$ be linear map with integer coefficients, i.e. there exists $M \in \mathbb{Z}^{n \times m}$ and $b \in \mathbb{Z}^m$ such that f(w) = Mw + b for each $w \in [0,1]^n$

Let $v = (x_1, \dots, x_n) \in [0, 1]^n$ be a rational vector. We define $\operatorname{den}(v)$ to be the least common denominator of $\{x_1, \dots, x_n\}$, i.e. the smallest $k \in \mathbb{Z}$ such that $kv \in \mathbb{Z}^n$.

Observe that

$$\operatorname{den}(v)f(v)=k(Mv+b)=M(\operatorname{den}(v)v)+\operatorname{den}(v)b\in\mathbb{Z}^m.$$

Then den(f(v)) is a divisor of den(v).

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The vector $\widetilde{v} = \operatorname{den}(v)(v, 1) \in \mathbb{Z}^{n+1}$ is called the homogeneous correspondent of v.

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Regular Simplexes

Regular Simplexes

The vector $\widetilde{v} = \operatorname{den}(v)(v, 1) \in \mathbb{Z}^{n+1}$ is called the homogeneous correspondent of v.

Definition

A simplex $S \subseteq [0,1]^n$ is called **regular** if the set of homogeneous correspondents of its vertices is part of a basis of the free abelian group \mathbb{Z}^{n+1} .

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Regular Simplexes

Regular Simplexes

Lemma

Let $S = \text{conv}(v_0, \dots, v_k) \subseteq [0, 1]^n$ be a regular k-simplex, and $\{w_0, \ldots, w_k\}$ a set of rational points in $[0, 1]^m$. Then the following conditions are equivalent:

- For each i = 1, ..., k, den (w_i) is a divisor of den (v_i) .
- (ii) For some integer matrix $M \in \mathbb{Z}^{n \times m}$ and integer vector $b \in \mathbb{Z}^m$, $Mv_i + b = w_i$.

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Regular Simplexes

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By a **regular triangulation** of a polyhedron *P* we understand a triangulation of *P* consisting of regular simplexes.

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Corollary

Let $P \subset [0,1]^n$ be a polyhedron, Δ be a regular triangulation of P and $f : \operatorname{ver}(\Delta) \to ([0,1] \cap \mathbb{Q})^m$ be such that $\operatorname{den}(f(v))$ divides $\operatorname{den}(v)$ for each $v \in \operatorname{ver}(\Delta)$. Then there exists a unique \mathbb{Z} -map $\eta : P \to [0,1]^m$ satisfying:

- 1. η is linear on each simplex of Δ ,
- 2. $\eta \upharpoonright \text{ver}(\Delta) = f$.

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Corollary

Let $P \subset [0,1]^n$ be a polyhedron, Δ be a regular triangulation of P and f : $ver(\Delta) \rightarrow ([0,1] \cap \mathbb{Q})^m$ be such that den(f(v)) divides den(v) for each $v \in ver(\Delta)$. Then there exists a unique \mathbb{Z} -map $\eta \colon P \to [0,1]^m$ satisfying:

- 1. η is linear on each simplex of Δ ,
- 2. $\eta \upharpoonright \text{ver}(\Delta) = f$.

Lemma

Let $\eta: P \to Q$ be a \mathbb{Z} -map there exists a regular triangulation Δ of P such that η is linear over each simplex in Δ .

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Regular Triangulations



Weighted Abstract Simplicial Complexes

For a regular triangulation Δ of a rational polyhedron P, the **skeleton** of Δ is weighted abstract simplicial complex

$$W_{\Delta} = (\mathcal{V}, \Sigma, \omega)$$

given by the following stipulations:

- 1. $\mathcal{V} = \text{vertices of } \Delta$.
- 2. For every subset $W = \{w_1, \dots, w_k\}$ of \mathcal{V} , $W \in \Sigma$ iff $conv(w_1, \dots, w_k) \in \Delta$.
- 3. $\omega \colon \mathcal{V} \to \mathbb{N}$, is defined by $\omega(\mathbf{v}) = \operatorname{den}(\mathbf{v})$

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Weighted Abstract Simplicial Complexes

Let $\mathfrak{W} = (\mathcal{V}, \Sigma, \omega)$ be a weighted abstract simplicial complex with vertex set $\mathcal{V} = \{v_1, \dots, v_n\}$.

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Weighted Abstract Simplicial Complexes

Let $\mathfrak{W} = (\mathcal{V}, \Sigma, \omega)$ be a weighted abstract simplicial complex with vertex set $\mathcal{V} = \{v_1, \dots, v_n\}$.

Let e_1, \ldots, e_n the standard basis vectors of \mathbb{R}^n , and $\Delta_{\mathfrak{W}}$ be the complex whose vertices are

$$\mathbf{v}_1' = \mathbf{e}_1/\omega(\mathbf{v}_1), \ldots, \mathbf{v}_n' = \mathbf{e}_n/\omega(\mathbf{v}_n),$$

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Weighted Abstract Simplicial Complexes

Let $\mathfrak{W} = (\mathcal{V}, \Sigma, \omega)$ be a weighted abstract simplicial complex with vertex set $\mathcal{V} = \{v_1, \dots, v_n\}$.

Let e_1, \ldots, e_n the standard basis vectors of \mathbb{R}^n , and $\Delta_{\mathfrak{W}}$ be the complex whose vertices are

$$\mathbf{v}_1' = \mathbf{e}_1/\omega(\mathbf{v}_1), \ldots, \mathbf{v}_n' = \mathbf{e}_n/\omega(\mathbf{v}_n),$$

and whose k-simplexes (k = 0, ..., n) are given by

$$\operatorname{conv}(v'_{i(0)}, \dots, v'_{i(k)}) \in \Delta_W \quad \text{iff} \quad \{v_{i(0)}, \dots, v_{i(k)}\} \in \Sigma.$$

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Weighted Abstract Simplicial Complexes

Let $\mathfrak{W} = (\mathcal{V}, \Sigma, \omega)$ be a weighted abstract simplicial complex with vertex set $\mathcal{V} = \{v_1, \dots, v_n\}$.

Let e_1, \ldots, e_n the standard basis vectors of \mathbb{R}^n , and $\Delta_{\mathfrak{W}}$ be the complex whose vertices are

$$\mathbf{v}_1' = \mathbf{e}_1/\omega(\mathbf{v}_1), \ldots, \mathbf{v}_n' = \mathbf{e}_n/\omega(\mathbf{v}_n),$$

and whose k-simplexes (k = 0, ..., n) are given by

$$\operatorname{conv}(v'_{i(0)},\ldots,v'_{i(k)}) \in \Delta_W \quad \text{iff} \quad \{v_{i(0)},\ldots,v_{i(k)}\} \in \Sigma.$$

Then $\Delta_{\mathfrak{W}}$ is a regular triangulation of the polyhedron $|\Delta_{\mathfrak{W}}| \subseteq [0,1]^n$ called the **canonical realization** of \mathfrak{W} .

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Given a weighted abstract simplicial complex $\mathfrak{W} = (\mathcal{V}, \Sigma, \omega)$.

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Weighted Abstract Simplicial Complexes

Given a weighted abstract simplicial complex $\mathfrak{W} = (\mathcal{V}, \Sigma, \omega)$.

The maps

$$\mathcal{V} \xrightarrow{f} [0,1]^n$$

such that den(f(v)) is a divisor of $\omega(v)$

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Weighted Abstract Simplicial Complexes

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The maps

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Weighted Abstract Simplicial Complexes

Given a weighted abstract simplicial complex $\mathfrak{W} = (\mathcal{V}, \Sigma, \omega)$.

The maps

$$\mathcal{V} \xrightarrow{f} [0,1]^n$$

such that den(f(v)) is a divisor of $\omega(v)$

are in one-one correspondence with

the \mathbb{Z} -maps

$$|\Delta_{\mathfrak{W}}| \xrightarrow{\eta} [0,1]^n$$

that are linear over each simplex of $\Delta_{\mathfrak{W}}$.

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Strongly Regular Polyhedra

Definition

A rational polyhedron P is said to be **strongly regular** if there is a regular triangulation Δ of P such that the denominators of the vertices of each maximal simplex of Δ are coprime.

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Strongly Regular Polyhedra

Definition

A rational polyhedron P is said to be strongly regular if there is a regular triangulation Δ of P such that the denominators of the vertices of each maximal simplex of Δ are coprime.

Lemma

A rational polyhedron P is strongly regular if and only if every regular triangulation Δ of P is such that the denominators of the vertices of each maximal simplex of Δ are coprime.

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Strongly Regular Polyhedra



Strongly Regular Polyhedra

Examples

- For every n = 1, 2, ... the *n*-dimensional cube $[0, 1]^n$ is strongly regular.
- ► Every regular n-simplex $S \subset [0, 1]^n$ is strongly regular.

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Theorem

Let P and Q be rational polyhedra and $\eta: P \to Q$ be a \mathbb{Z} -morphism onto Q. If P is a strongly regular then Q is strongly regular.

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Theorem

Given a polyhedron $P \subseteq [0,1]^n$ the following conditions are equivalent:

- (a) There exist m and a \mathbb{Z} -map $\eta: [0,1]^m \to P$ onto P.
- (b) P satisfies the following conditions:
 - 1. P is connected.
 - 2. $P \cap \{0,1\}^n \neq \emptyset$, and
 - 3. P is strongly regular.

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First Step: Strongly Regular Collapsible Polyhedra

Given an abstract simplicial complex $\langle V, \Sigma \rangle$ a simplex $T \in \Sigma$ is said to have a *free face F* if $\emptyset \neq F \subseteq T$ is a facet of T, and if $F \subseteq S \in \Sigma$ then S = F or S = T.

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The transition from $\langle V, \Sigma \rangle$ to the subcomplex $\langle V', \Sigma' = \Sigma \setminus \{T, F\} \rangle$ of $\langle V, \Sigma \rangle$, where $V' = V \setminus F$ if F is a singleton and otherwise V' = V is called an *(abtract) elementary collapse*

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We say that $\langle V, \Sigma \rangle$ is **collapsible** if it collapses to the abstract simplicial complex consisting of one of its vertices (equivalently any of its vertices).

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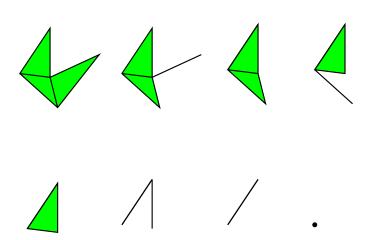
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First Step: Strongly Regular Collapsible Polyhedra

Theorem (Mundici & LMC)

Let $P \subseteq [0,1]^n$ be a polyhedron. Suppose

- (i) P has a collapsible triangulation ∇ ;
- (ii) P contains a vertex v of [0, 1]ⁿ;
- (iii) P is strongly regular.

Then there is a \mathbb{Z} -map $\eta: [0,1]^n \to P$ onto P.

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First Step: Strongly Regular Collapsible Polyhedra

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- (i) P has a collapsible triangulation ∇ ;
- (ii) P contains a vertex v of $[0, 1]^n$;
- (iii) P is strongly regular.

Then P is a \mathbb{Z} -retract of $[0,1]^n$.

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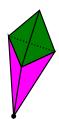
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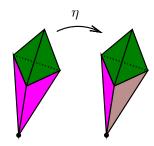
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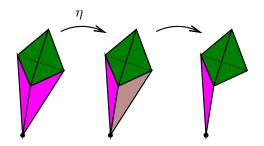
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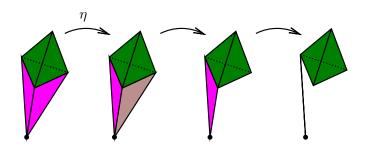
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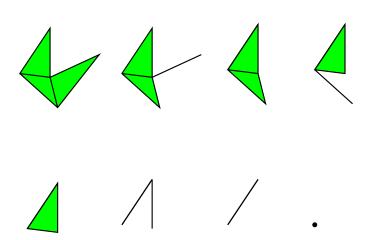
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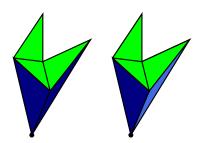
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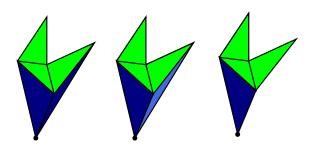
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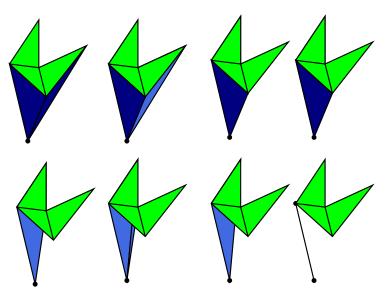
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Let $P \subseteq [0,1]^n$ be a rational polyhedron satisfying:

- 1. P is connected,
- 2. $P \cap \{0,1\}^n \neq \emptyset$, and
- 3. P is strongly regular.

Let Δ be a regular triangulation of P.

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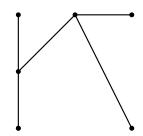
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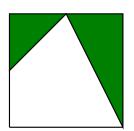
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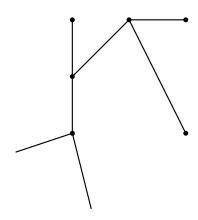
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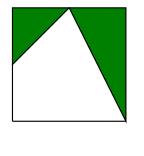
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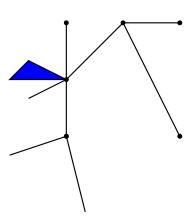
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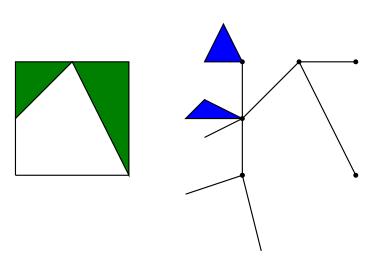
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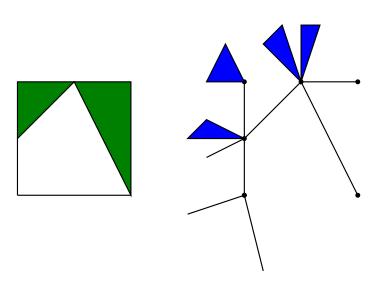
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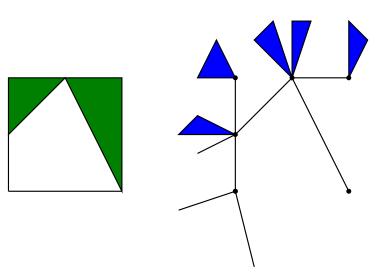
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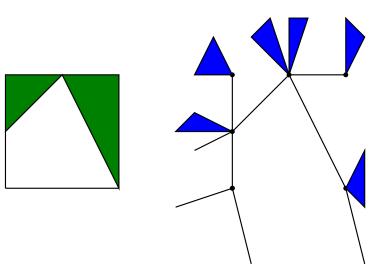
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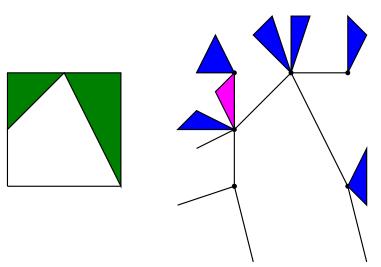
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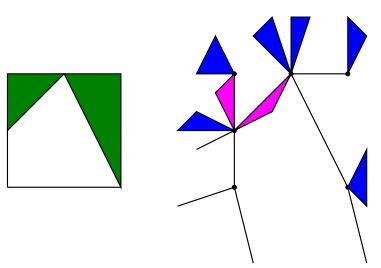
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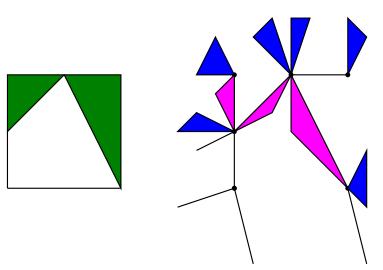
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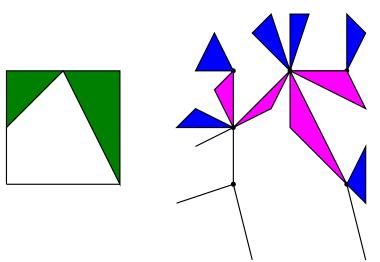
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Weakly Projective MV-algebras

Definition

An MV-algebra A is said to be (finitely generated) **weakly projective** if there exist positive integers m and n, and homomorphisms $f: \mathcal{M}([0,1]^m) \to A$ and $g: A \to \mathcal{M}([0,1]^n)$, such that f is onto A and g is one-one.

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Theorem

An MV-algebra is weakly projective if and only if there exist n and a rational polyhedron $P \subseteq [0,1]^n$ satisfying the following conditions:

- 1. $A \cong \mathcal{M}(P)$,
- 2. $P \cap \{0,1\}^n \neq \emptyset$,
- 3. P is connected, and
- 4. P is a strongly regular.

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An MV-algebra A is weakly projective if and only if it is finitely presented and $A \in \mathbb{IS}(\mathcal{M}([0,1]^n))$ for some n.

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An MV-algebra A is weakly projective if and only if it is finitely presented and $A \in \mathbb{IS}(\operatorname{Free}_{MV}(\omega))$.

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If a pair of formulas φ, ψ with variables $\{x_1, \dots, x_n\}$ are such that $\mathcal{M}([0,1]^n)/\text{cong}(f_{\varphi}, f_{\psi}) \in \mathbb{ISP}_U(\text{Free}_{MV}(\omega))$,

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If a pair of formulas φ, ψ with variables $\{x_1, \ldots, x_n\}$ are such that $\mathcal{M}([0,1]^n)/\mathrm{cong}(f_{\varphi}, f_{\psi}) \in \mathbb{ISP}_U(\mathrm{Free}_{MV}(\omega))$, then:

$$\varphi \approx \psi \models_{\mathsf{Free}_{MV}(\omega)} \{\alpha_i \approx \beta_i \mid i = 1, \dots, m\}$$

if and only if there exist $j \in \{1, ..., m\}$ such that

$$\varphi \approx \psi \models_{MV} \alpha_j \approx \beta_j$$

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If a pair of formulas φ, ψ with variables $\{x_1, \ldots, x_n\}$ are such that $\mathcal{M}([0,1]^n)/\mathrm{cong}(f_{\varphi}, f_{\psi}) \in \mathbb{ISP}_U(\mathrm{Free}_{MV}(\omega))$, then:

$$\varphi \approx \psi \models_{\mathsf{Free}_{MV}(\omega)} \{\alpha_i \approx \beta_i \mid i = 1, \ldots, m\}$$

if and only if there exist $j \in \{1, ..., m\}$ such that

$$\varphi \approx \psi \models_{MV} \alpha_j \approx \beta_j$$

if and only if

$$\varphi \leftrightarrow \psi \vdash_{\mathbf{L}_{\infty}} \alpha_{\mathbf{j}} \leftrightarrow \beta_{\mathbf{j}}$$

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If a formula φ with variables $\{x_1,\ldots,x_n\}$ is such that $\mathcal{M}([0,1]^n)/\mathrm{cong}(f_{\varphi},1)\in\mathbb{ISP}_U(\mathrm{Free}_{MV}(\omega))$, then:

$$\varphi \approx \varphi \rightarrow \varphi \models_{\mathsf{Free}_{\mathsf{MV}}(\omega)} \{\alpha_i \approx \alpha_i \rightarrow \alpha_i \mid i = 1, \ldots, m\}$$

if and only if there exist $j \in \{1, ..., m\}$ such that

$$\varphi \approx \varphi \rightarrow \varphi \models_{MV} \alpha_j \approx \alpha_j \rightarrow \alpha_j$$

if and only if

$$\varphi \vdash_{\mathbf{L}_{\infty}} \alpha_{\mathbf{j}}$$

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[E. Jeřábek, Bases of Admissible Rules of Łukasiewicz Logic, *Journal of Logic Computation* 20 (6), 1149-1163 (2010)]

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Admissible Saturated



Admissible Saturated Formulas

[E. Jeřábek, Bases of Admissible Rules of Łukasiewicz Logic, *Journal of Logic Computation* 20 (6), 1149-1163 (2010)]

Definition

A formula φ is admissibly saturated in a logic L if for every finite set Δ of formulas, $\varphi \hspace{0.2em}\sim_{\hspace{-0.9em} L} \Delta$ implies $\varphi \hspace{0.2em}\sim_{\hspace{-0.9em} L} \psi$ for some $\psi \in \Delta$.

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If φ is a formula such that $\mathfrak{M}([0,1]^n)/\mathrm{cong}(f_{\varphi},1)$ is weakly projective, then φ is admissible saturated (in \mathbb{L}_{∞}).

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Admissible Saturated Formulas

If φ is a formula such that $\mathfrak{M}([0,1]^n)/\mathrm{cong}(f_{\varphi},1)$ is weakly projective, then φ is admissible saturated (in \mathbb{L}_{∞}).

Theorem (E. Jeřábek)

A formula φ with variables in $\{x_1, \ldots, x_n\}$ is admissibly saturated in \mathcal{L}_{∞} if and only if

- (i) $f_{\omega}^{-1}(1) \cap \{0,1\}^n \neq \emptyset$,
- (ii) $f_{\varphi}^{-1}(1)$ is connected, and
- (iii) $f_{\omega}^{-1}(1)$ is a finite union of anchored polytopes.

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Theorem

And MV-algebra is weakly projective if and only if there exist n and a rational polyhedron $P \subseteq [0,1]^n$ satisfying the following conditions:

- 1. $A \cong \mathcal{M}(P)$,
- 2. $P \cap \{0,1\}^n \neq \emptyset$,
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A formula φ with variables in $\{x_1, \ldots, x_n\}$ is admissibly saturated in \mathbb{L}_{∞}

if and only if

 $\mathcal{M}([0,1]^n)/\text{cong}(f_{\varphi},1)$ is weakly projective.

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- ► Images of Z-maps.
- Weakly projective MV-Algebras.

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- ► Images of Z-maps.
- Weakly projective MV-Algebras.
- Relation with admissible saturated formulas.

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Thank you!

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